A Computational Approach to Competition Impact Assessment

by

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ABSTRACT

Competition does not take place in a vacuum but is embedded within an existing legal and regulatory environment. Competition authorities are thus encouraged to evaluate existing laws to identify and remediate competition effects. To this end, organizations such as the OECD and the World Bank have released guidance on the conduct of competition impact assessments. Despite the importance and complexities of a competition impact assessment, the literature is sparse when it comes to implementation specifics. From selection of laws to be reviewed to the actual assessment of legal provisions, much is left to the subjective evaluation of assessors. This could mean that errors could compound in the course of analysis and lead to implausible results. For example: 1. There are no parameters for law selection that aligns with market definitions; 2. There is no consistent, granular unit of analysis; 3. There is a lack of provable basis for attributing specific competition effects to legal texts. The work aims to apply techniques of computational law to these problems. First: The work will encode the norms applicable to a specific sector (digital payments market in the Philippines) into forms that are amenable to computational processing – such as modelling market entities and interactions into a knowledge graph, and the normative constraints into inference rules. Second, these representations can then be subjected to automated reasoning in order to provide insights useful to competition analysis, such as: determination of other relevant laws, evaluation for consistency and compliance, and proving of specific competition effects.

Thesis supervisor: Edward C. Pickering

Title: Professor of Physics

Acknowledgments

Write your acknowledgments here.

Biographical Sketch

Silas Whitcomb Holman was born in Harvard, Massachusetts on January 20, 1856. He received his S.B. degree in Physics from MIT in 1876, and then joined the MIT Department of Physics as an Assistant. He became Instructor in Physics in 1880, Assistant Professor in 1882, Associate Professor in 1885, and Full Professor in 1893. Throughout this period, he struggled with increasingly severe rheumatoid arthritis. At length, he was defeated, becoming Professor Emeritus in 1897 and dying on April 1, 1900.

Holman's light burned brilliantly before his tragic and untimely death. He published extensively in thermal physics, and authored textbooks on precision measurement, fundamental mechanics, and other subjects. He established the original Heat Measurements Laboratory. Holman was a much admired teacher among both his students and his colleagues. The reports of his department and of the Institute itself refer to him frequently in the 1880's and 1890's, in tones that gradually shift from the greatest respect to the deepest sympathy.

Holman was a student of Professor Edward C. Pickering, then head of the Physics department. Holman himself became second in command of Physics, under Professor Charles R. Cross, some years later. Among Holman's students, several went on to distinguish themselves, including: the astronomer George E. Hale ('90) who organized the Yerkes and Mt. Wilson observatories and who designed the 200 inch telescope on Mt. Palomar; Charles G. Abbot ('94), also an astrophysicist and later Secretary of the Smithsonian Institution; and George K. Burgess ('96), later Director of the Bureau of Standards.

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Chapter 1

Introduction

1.1 Competition and the Legal Environment

Competition does not take place in a vacuum but is embedded within an existing legal and regulatory environment.[1] Barriers to entry and exit (which can cause failure to produce a competitive market) can arise not just from the structural features of a market, or the behavior of its actors, but also the policy environment maintained by the government.[2] The law can affect competition in a number of ways: It can openly favor some players, providing them with tax exemptions and subsidies. It can also put other players at a disadvantage, by making it more expensive for them to operate in the industry (through barriers to entry and exit). In both of the above cases, the law works would have the impact of limiting competition through advantages and constraints directly addressed to industry players. The enactment of a competition policy, specifically, anti-trust law, is explicitly directed at the competitive behavior of firms, and is designed to restrain monopoly and maintain market competition.

[1] This evokes the vision of law not as a neutral, static stage, but as Laurence Tribe describes it - one possessed of a curvature, a shape, that can affect the movement of the actors on it. See Generally Laurence H. Tribe. "The Curvature of Constitutional Space: What Lawyers Can Learn from Modern Physics". In: *Harvard Law Review* (Nov. 1989), pp. 1–68.

[2] See Erlinda M Medalla. "Understanding the New Philippine Competition Act". In: *Philippine Institute for Development Studies (PIDS) Discussion Paper Series* (No. 2017-14 2017), pp. 1–24. URL: http://hdl.handle.net/10419/173591 (visited on 01/08/2024), at 5.

However, even the general legal environment outside of competition law can also work against competition through more subtle mechanisms. The law can control the flow of information between and amongst buyers and sellers, constraining their strategic choices. More importantly, the law can allow the state itself, with its size, economic power, and monopoly on regulatory powers, to be a direct player (as a buyer or seller) in any industry.[3]

1.2 Competition Impact Assessments

The law's impact on competition underscores the need for detailed studies on how the current legal and regulatory backdrop affects competition. This can be performed through a **competition impact assessment** of the laws that operate in specific sectors of economic activity. A competition impact assessment refers to a systematic, evidence-based process of reviewing existing or proposed policies in order to determine their impact on competition. This is performed with the view to formulating alternative policies that are more conducive to competition.[4] The underlying logic is that while governments may pursue important policy goals through legislation - there are multiple pathways to these goals, and governments should pursue those paths that least impact competition. This in

[3] In some cases, the general legal environment is just us important as the competition law in the maintenance of a competitive market. See generally Iftekhar Hasan and Matej Marinč. "Should Competition Policy in Banking Be Amended during Crises? Lessons from the EU". in: *European Journal of Law and Economics* 42.2 (Oct. 2016), pp. 295–324. ISSN: 0929-1261, 1572-9990. DOI: 10.1007/s10657-013-9391-2. URL: http://link.springer.com/10.1007/s10657-013-9391-2 (visited on 01/08/2024), which suggests that competition policy in the financial sector can be inconsistent in times of crisis. Financial regulators, through prudential standards, bear the greater responsibility in ensuring against concentration. To the extent that this overlaps with market structure concerns of competition authorities, greater coordination is required. See also Tomaso Duso, Jo Seldeslachts, and Florian Szücs. "The Impact of Competition Policy Enforcement on the Functioning of EU Energy Markets". In: *The Energy Journal* 40.5 (Sept. 2019), pp. 97–120. ISSN: 0195-6574, 1944-9089. DOI: 10.5547/01956574.40.5.tdus. URL: http://journals.sagepub.com/doi/10.5547/01956574.40.5.tdus (visited on 01/08/2024), Competition policy may have significant impacts, but only to the lightly regulated sectors. On the other hand, highly-regulated firms are less likely to respond to competition policy.

[4] Nakaizumi defines Impact Assessment as "a process that prepares evidence for political decision makers on the advantage and disadvantage of possible regulatory options by assessing their potential impacts" and Competition Assessment as a form of Impact Assessment that assesses "whether the regulation unnecessarily restricts competition or not". Takuya Nakaizumi. *Impact Assessment for Developing Countries: A Guide for Government Officials and Public Servants*. Contributions to Economics. Singapore: Springer Nature Singapore, 2022. ISBN: 978-981-19549-3-1 978-981-19549-4-8. DOI: 10.1007/978-981-19-5494-8. URL: https://link.springer.com/10.1007/978-981-19-5494-8 (visited on 07/14/2024), at 3; An impact assessment refers to both the textitex-ante evaluative process as well as the output of that process Nakaizumi, *Impact Assessment for Developing Countries*, at 6.

turn springs from the premise that more competition is beneficial, especially for the consumers.[5] In addition to isolating and predicting the effect of laws, a competition impact assessment can lead to greater transparency in policymaking, by disclosing the law's interactions with other commitments, as well as providing a venue for experts and other stakeholders to participate.[6]

The Philippine Competition Commission (PCC) has already conducted several such assessments of selected laws[7] - either at its own instance or upon request by Congress or regulatory agencies. It has also worked with organizations such as the OECD, which has performed competition impact assessments of certain economic sectors.[8] The OECD and other organizations interested in advocating for competition policy have also issued guidelines for the conduct of competition impact assessments.[9] The PCC currently has unpublished draft guidelines[10] that it uses to guide its competition assessment exercises. The PCC Guidelines disclose that it is based on the OECD Guidelines as well as the World Bank's Markets and Competition policy Assessment Toolkit.[11]

[5]OECD. *Competition Assessment Toolkit - Volume 1 (Principles)*. 2019. URL: https://www.oecd.org/daf/competition/46193173.pdf (visited on 10/10/2023), at 7.

[6]Nakaizumi, *Impact Assessment for Developing Countries*, at 6-7.

[7]The methodology and findings of the PCC for the conduct of the assessment will form part of the knowledge domain to be encoded, see Chapter 4 below. See also the discussion below on definition of sectors, criteria for selecting laws, and units of analysis.

[8]See for example OECD. *Competition Assessment Reviews: Logistics Sector in the Philippines*. 2020. URL: https://www.oecd.org/daf/competition/oecd-competition-assessment-reviews-philippines-2020.pdf (visited on 10/10/2023); See also OECD. *Competitive Neutrality Reviews: Small-Package Delivery Services in the Philippines*. 2020. URL: https://www.oecd.org/daf/competition/oecd-competitive-neutrality-reviews-philippines-2020.pdf (visited on 10/10/2023).

[9]See OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, for Part 1 of the Organization for Economic Cooperation and Development's (OECD) 3-part guidelines for competition impact assessment. This and subsequent volumes will be referred to collectively as the "OECD Guidelines"; See OECD Guidelines, supra. See also the International Competition Network's (ICN) recommended practices. Subsequently referred to as the "ICN Guidelines" ICN Advocacy Working Group. *Recommended Practices on Competition Assessments*. International Competition Network, 2014. URL: https://www.internationalcompetitionnetwork.org/wp-content/uploads/2018/07/AWG_RP_English.pdf (visited on 01/10/2024).

[10]"PCC Guidelines". On file with the author. The PCC has not released consolidated guidelines on the conduct of competition impact assessments, but it has referenced the OECD Guidelines and other methodological bases in its extant assessment exercises, which are contained in its periodic issue papers and market analyses, to signal its sense of a particular economic sector and guide its enforcement priorities. These papers are available in its website at https://www.phcc.gov.ph/resource-category/issues-papers.

[11]See The World Bank. *Markets and Competition Policy*. World Bank. URL: https://www.worldbank.org/en/topic/competition-policy (visited on 01/16/2024), Subsequently, "the World Bank Assessment Toolkit". The OECD Guidelines will be discussed in more detail later. Full documentation on the World Bank Assessment Toolkit is not publicly available.

To the extent that these guidelines and instances of their implementation converge into common methodology, these guidelines will be idealized into a "canonical approach" to competition impact assessment, and represented by the OECD Guidelines as the focus of analysis.

1.3 The Canonical Approach to Competition Impact Assessment

The canonical approach to conducting competition assessments starts with identifying laws that are relevant to a sector, then proceeds to evaluating such laws for competition effects.As a subset of the broader process of impact assessment, a competition assessment would need to first identify the problem at stake. The problem would often be the need to reconcile measures with the norm that is relevant to the evaluator (e.g. environmental protection in the case of an environmental impact assessment, or a competitive market for competition impact assessments). Comparative impacts of the measures are identified, along with their relative advantages and disadvantages relative to the achievement of some goal. This awareness of the underlying goal allows for the generation of alternative measures that can achieve the goal while mitigating the impact against the relevant norm.[12]

As elaborated in the OECD Guidelines, the process of competition impact assessment involves the following steps:[13]

1. **Focus on a relevant sector** The OECD Guidelines assume that a competition impact assessment will not cover the economy as a whole but will focus on a particular sector, i.e. a set of related products or services. Factors related to the choice of sector will be discussed in a later chapter. However, the work will proceed under the assumption that the definition and choice of a sector will not be a computational problem.

[12]Nakaizumi, *Impact Assessment for Developing Countries*, at 7.

[13]The enumerated steps are from OECD. *Competition Assessment Toolkit - Volume 3 (Operations Manual)*. 2019. URL: https://web-archive.oecd.org/2020-01-22/370055-COMP_Toolkit_Vol.3_ENG_2019.pdf (visited on 10/10/2023), at 14-15.

2. **Identify the laws to be assessed** - This can be straightforward in the case of assessing new or pending legislation or regulation. On the other hand, for situations where the impact of laws on an entire economic sector is required, discretion is involved in defining the boundaries of what will be reviewed. This is expected to result in a list of relevant laws.

3. **Apply threshold tests** - The list of relevant laws can be narrowed down through a threshold test. This is based on a checklist of questions designed to identify potential restrictions to competition. This will result in a smaller set of flagged laws that can be subject to a more detailed review.

4. **Detailed review of flagged laws** - Performing a more detailed review to determine whether or not there are "actual and significant" restrictions on competition. Those with such restrictions form a set of "critical laws" for which the next stage of the process should be applied.

5. **Generate alternatives** - For those critical laws where restrictions are found, identify alternative measures that can achieve policy objectives while being less restrictive or competition.

6. **Selecting the best option** - Once policy alternatives have been generated, a judgment must be determined as to the “best" option. Once the "best" option has been identified, legislation must be drafted and passed that will implement this policy recommendation.

7. **Ex-post assessment** - Review and monitoring of the impacts of the law implementing the selected policy alternative.

The canonical approach requires a search methodology to enumerate the laws that can apply to the actors and transactions in a given market. The documentation assumes that the government will select or prioritize a sector to be assessed. The guidelines suggests a number of prioritization principles to aid in this determination, such as: 1. Selecting sectors with high economic impact (in terms of share of GDP, consumer expenditure, employment); 2. Or those have been the subject of frequent complaints or interventions; 3. The constraints of time, financial resources, and the

availability of technical talent.[14] For the purpose of this work, it will be assumed that selection and prioritization can proceed independently, prior to the methodology to be outlined in this work. Aside from simplifying the scope of the work, the assumption is compatible with the notion that selection and prioritization of the sector is a matter of policy, to be made by accountable, human institutions.

Once a sector has been selected, the next step is to compile legislation that is relevant to the sector. This, in turn, is predicated on delineating a conceptual boundary for the sector. The guideline acknowledges that a boundary-setting exercise can be challenging.[15] To provide some structure into this exercise, the guidelines provide some suggestions on how to proceed: 1. Focusing on legislation relevant to one ministry. Using the correlation with the ministry concerned as a proxy for a relevant boundary, however, simply restates the problem especially where the ministry has a broad mandate. It can also risk missing laws that require inter-agency coordination; 2. Focusing on standard definitions - This can be done by referring to standard industry classifications, such as the United Nation's International Standard Industrial Classification of All Economic Activities,[16] or the Statistical Classification of Economic Activities in the European Union.[17] The guideline, however, notes that these classification systems will often segregate economic activities in ways that are counter to both intuition as well as grounded knowledge as to how industries are actually run.

Assuming that the boundary of a market sector can be defined for purposes of finding relevant laws - this process may still yield numerous laws for any modern regulatory environment.For this, the canonical approach suggests a process for filtering relevant laws in order to arrive at a set of critical laws.[18]

[14] OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*, at 18-19.

[15] The OECD guidelines refer to the need to evaluate competition impact on sectors, without providing a method for identifying and delineating sectors. The question may ultimately be considered as one of policy instead of law, and will be beyond the scope of this work.The presence of a law that addresses a particular set of economic actors as well their transactions is a good signal that the policymakers have recognized it as a sector.

[16] United Nations. *International Standard Industrial Classification of All Economic Activities (ISIC)*. Revision 4. United Nations, 2008. ISBN: 978-92-1-161518-0. URL: https://unstats.un.org/unsd/publication/seriesm/seriesm_4rev4e.pdf.

[17] European Commission. *Statistical Classification of Economic Activities in the European Union*. Rev. 2. 2008. URL: https://ec.europa.eu/eurostat/documents/3859598/5902521/KS-RA-07-015-EN.PDF (visited on 05/13/2024).

[18] OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*, at 17.

The critical laws are then analyzed and evaluated for their impact on the competition environment. The analysis is coupled with the generation of alternative policies, for evaluation as to their comparative impact on competition (relative to the current rule).

While there is no standard methodology for this evaluation stage, the OECD guidelines and previous impact assessment exercises put an emphasis on economic counterfactual reasoning.Counterfactial reasoning is usually not used during the threshold testing phase but in the ex-post assessment phase of competition impact assessment. This is to determine if competition is in fact optimized after the removal of the anti-competitive law.[19]

1.4 Problems with the Canonical Approach

Despite the importance of evaluating the competition impact of the legal environment, the methodological tool set for impact assessment has fallen behind (in terms of sophistication and rigor) compared to those used in other areas of competition policy, such as: merger control, assessment of anticompetitive agreements and abuse of dominance.[20] This gives rise to problems in the selection of laws as well as threshold analysis.

1.4.1 Specific Problems with Sector Definition and Relevant Law Selection

The first step in the canonical approach to competition impact assessment is to find the laws that are applicable to a sector. Given an industry or market sector under consideration - What law "covers" an industry or a market with all its actors and transactions? The goal is to arrive at either a complete or a heuristic but consistent mapping between actors (and their actions relevant to a market) - and the laws that would cover these actors and actions.

[19] Ibid., at 123-124.

[20] See Nicole Robins and Hannes Geldof. "Ex Post Assessment of the Impact of State Aid on Competition". In: *European State Aid Law Quarterly* 17.4 (2018), pp. 494–508. ISSN: 16195272, 21908184. DOI: 10.21552/estal/2018/4/6. URL: http://estal.lexxion.eu/article/ESTAL/2018/4/6 (visited on 01/03/2024), which proposes a greater role for financial and economic analysis in evaluating the impact of state action on competition, at 494-495.

Although no strict definition of sector and selection criteria for relevant laws are specified, some guidance may be availed from recent competition assessment exercises conducted by the PCC.[21] In defining the "cargo services industry" sector, for example, the PCC relied on an internationally recognized model for the structure and components of the logistics industry - one that encompasses not just movement of materials, but also related information management, inventory, warehousing, and liaising for compliance with border, regulatory procedures.[22] The PCC also relied on self-reporting of entities involved in the sector, and their referral of other members of their industry. This converged with a definition of the sector derived from data collected by the Philippine Statistics Authority, which maintains an index of industrial and commercial activities for organizing economic data collection.[23] A similar approach was taken in the PCC's analysis of the the milk industry - which was defined by grouping together products that share related codes under the Philippine Standardized Industrial Code, used by the Philippine Statistics Authority to keep track of economic activity (which is in turn largely based on self-identification of the players involved).[24]

Tracing through the supply chain of a product or entity known to be part of the sector can also be used to expand from specific cases into a defined sector. By identifying products and services along the supply chain, the PCC was able to define and the sector and focus on relevant laws for the downstream petroleum industry,[25]. In the case of defining the social media and search markets, the PCC began with a recognized academic definition, known actors (e.g. Meta, Google) and then extrapolated the rest of the market structure through market interactions (partnerships, contracts,

[21]The PCC's competition impact assessment work (its methodologies and findings) are contained in periodic Market Studies and Issues papers, available through its website - https://www.phcc.gov.ph/resource-category/issues-papers. This body of work is considered the baseline for reckoning improvements from a computational approach, as well as a source of interpretive rules to be encoded.

[22]Gilberto M. Llanto and Francis Mark A. Quimba. "Competition Policy Issues in Cargo Services". In: *PCC Issues Paper* 04 (2021). URL: https://www.phcc.gov.ph/storage/pdf-resources/1678085340_PCC-Issues-Paper-2021-04-Competition-Policy-Issues-in-Cargo-Services.pdf, at 12.

[23]Ibid., at 8-9.

[24]Francis Mark A. Quimba and Kirsten J. Dela Cruz. "Issues Paper on the Philippine Milk Products Industry". In: *PCC Issues Paper* 3 (2021). URL: https://www.phcc.gov.ph/storage/pdf-resources/1678085617_PCC-Issues-Paper-2021-03-Issues-Paper-on-the-Philippine-Milk-Products-Industry.pdf.

[25]Peter L. U et al. "Market Study on the Refined Petroleum Industry". In: *PCC Issues Paper* 05 (2021). URL: https://www.phcc.gov.ph/storage/pdf-resources/1678085292_PCC-Issues-Paper-2021-05-Market-Study-on-the-Refined-Petroleum-Industry.pdf.

rivalries, etc).[26]

Despite these efforts at a "principled" definition based on components of the market (i.e. products and services offered, participants, supply chains), what often happens is that the sectors and relevant laws are determined *a priori*, through a process that is outside of the competition impact assessment itself.

Often, the pattern seems to be that the designation of sectors and the selection of relevant laws are driven by current (if transient) constituent concerns (high prices, abusive conduct) that may be addressed by an existing law, but now to be reviewed for possible competition issues that may, over the long term, harm consumer welfare. Such was the case of the PCC's assessment of the effects of the Mobile Number Portability (MNP) Act, where the PCC states that the law was selected because telecommunications was a priority sector at tie time of publication, and that the law was passed based on the promise that it would spur competition in the Philippine telecommunications market.[27] The PCC's view of the scope of a sector can also be shaped by its annual investigation and enforcement agenda, which can be determined in coordination with other government agencies. This may also inform the list of laws to be assessed, since these agencies already have their portfolio of laws to enforce, which they wish to assess for competition impact. In the case of the social media and search, the PCC identified known or self-reported regulators of the market involved, and then proceeded with a network search of the agency's legal mandates for the market.[28]

Assuming that the economic sector has been selected and its conceptual boundaries have been delineated, the assessor is expected to derive from this model "an exhaustive list of laws and

[26] Katrielle Pauline C Abrenica, Gwyneth Mae R Ramos, and Marciano D Cleofas III. "Competition Policy and Data Privacy in Social Media and Search Platforms". In: *PCC Market Study* 01 (2025). URL: https://www.phcc.gov.ph/storage/pdf-resources/1768987578_1768981498_PCC%20Market%20Study%20No.%2001,%20Series%20of%202025%20Competition%20Policy%20and%20Data%20Privacy%20in%20Social%20Media%20and%20Search%20Platforms-compressed.pdf, at 6.

[27] Jestoni A Olivo, Jacob John A Flores, and Israel D Bañez. "Competition Impact Assessment of the Mobile Number Portability Act (RA No. 11202)". In: *PCC Market Study* 02 (2025). URL: https://www.phcc.gov.ph/storage/pdf-resources/1770368318_PCC%20Market%20Study%20No.%202-Competition%20Impact%20Assessment%20of%20the%20Mobile%20Number%20Portability%20Act%20(RA%20No.%2011202)-revised-02062026.pdf, at 5.

[28] Abrenica, Ramos, and Cleofas III, "Competition Policy and Data Privacy in Social Media and Search Platforms".

regulations that influence the economic activities that take place in each of the sectors under examination".[29] There is no elaboration as to how the conceptual mapping in the previous step can translate into a search strategy that can be documented, refined, and shared. There are no parameters for law selection that aligns with market definitions. The literature require that laws relevant to a market be subject to competition analysis. Although competition law offers well-established methods for defining the relevant market,[30] criteria for selecting laws that will be subject to analysis are not aligned with these market definitions. Thus, assessors are likely to under- or over-select the laws to be included, or provide no cogent explanation as to why any specific law should or should not be included in the analysis.

Risk of under-inclusion In evaluating a legal environment, assessors may overlook critical legal standards pertinent to the industry, due to a disconnect in the terms used by the industry and those employed in legal contexts. This phenomenon is particularly evident in emerging digital finance businesses. These entities often operate under novel designations or through distinct modalities, such as exchanges and applications. Consequently, there is a prevailing misconception that such entities fall outside the purview of traditional financial regulations. This overlooks the fact that, despite their innovative approaches, these entities perform analogous functions and are subject to similar risks as their conventional counterparts.

On the other hand, focusing on a single law nominated by a stakeholder may result in losing many critical signals. This is because competitive issues such as barriers to entry, disproportionate costs, and preferential treatment can arise not from a single law but from the interaction of key provisions spread across several legislative enactments.

[29] The Guidelines acknowledge that this stage of the process is not trivial, since ensuring the inclusion of all relevant legislation requires casting a broad net. The guideline suggests an iterative process, which requires not just reference to texts initially found in electronic databases (as well as the laws they refer to, such as implementing rules), but also through consultation with stakeholders. OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*, at 21.

[30] See for example the discussion in Chapter 4, on the Small but Significant and Non-transitory Increase in Price (SSNIP) test used for defining the relevant market

The OECD framework cautions that "When performing this exercise, it is important to remember that, in addition to sector-specific regulation, there also exists horizontal, cross-sectoral, legislation (such as planning restrictions or environmental standards) that may have a considerable impact on the economic activities performed in that sector and may be a cause of additional competition restrictions."[31]

Risk of over-inclusion If we take a connected view of the law, all of the law can be relevant to a particular market. Every law has the potential to change the relative rights and obligations of parties involved in economic activity and thus result in some competition impact. Considered this way, even laws that apply to everyone and only incidentally touch economic activities in a sector - can be interrogated for potential competitive impact, no matter how small or contingent. The Philippine Civil Code, for example, gives a preferential status for unsecured debt evidenced by a notarized instrument, over those that are embodied in a private document. Family and tax law provides rights and privileges that are accessible only to married heterosexuals. Nevertheless, these laws should usually not be the subject of competition impact assessment. As they apply to everyone, in a large enough population their application to specific individuals can appear random and evenly distributed - any *de minimis* competitive impact is contingent and cancel each other out. More importantly, these laws do not directly relate to the sector under consideration, and their ultimate effects on the actors in the sector will only be coincidental.[32]

One dilemma in connecting a relevant law to a sector, is the fundamental question of whether or not the law is addressed specifically to regulate the sector, as legislation that we need to look at in economic/competition terms or is it an addressed to other more fundamental concerns, e.g.

[31] See OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*.

[32] "Plaintiffs stress that the LMRDA is a remedial measure and seek a liberal construction. This maxim is useless in deciding concrete cases. Every statute is remedial in the sense that it alters the law or favors one group over another... But after we determine that a law favors some group, the question becomes: How much does it favor them? Knowing that a law is remedial does not tell a court how far to go. Every statute has a stopping point, beyond which, Congress concluded, the costs of doing more are excessive — or beyond which the interest groups opposed to the law were able to block further progress." - Richard Stomper, et al., Plaintiffs-appellees, v. Amalgamated Transit Union, Local 241, Defendant-appellant, 27 F.3d 316 (7th Cir. 1994)

for environment, for safety, consumer welfare, etc. Of course, in any advanced legal system in a connected economy, any law can impose some cost that can cascade to an economic impact on market participants, which can affect their competitive behavior.

A lawyer's theoretical training and experience, and openness to economic thinking can perform the required analysis while compensating for these limitations. However, that lawyer may not always be available. It is also possible that the limited pool of legal talent will not be able to scale to the demands of extensive, industry-wide competition analysis, which could involve hundreds (if not thousands) of laws and regulations, all of which could interact with each other. Abstracting the problem into a computational form can allow parts of the analysis to be done by non-lawyers (i.e., the staff of a competition authority), or even by computers. The goal is not to supplant the human component of competition impact assessment but to augment it.

The work assumes that a sector of economic activity has been defined, in either ad hoc or through some textually expressed standard. As will be discussed in a later chapter, the relevance and sector definition problem with distinct analytical requirements from the normative evaluation performed in this work.

1.4.2 Specific Problem with Threshold Testing

According to the OECD Guidelines, Step 3 (the threshold test) and the associated competition checklist lie at the heart of the competition impact assessment process. Despite the importance assigned to this section of the process, both the OECD Guidelines and the literature on competition impact assessment do not provide a detailed, rigorous, and consistent methodology for performing the threshold testing step.Throughout the three volumes of the OECD Guidelines, instead of specifying the competition norms in terms of specific rules, the norms are surfaced through examples. Threshold testing inherently assumes a defined relevant market - The OECD Guideline often treat these as givens for human evaluators.[33] If we are to automate at least part of the process, we cannot

[33] See generally OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*.

assume any unstated context. In order to evaluate whether a rule facilitates "abuse of dominant position" or creates a barrier to entry, the system will require an explicit ontological model of the market itself.

The OECD Guidelines provide a checklist of questions that can be used to identify potential restrictions to competition. However, the OECD Guidelines do not provide a detailed methodology for applying the checklist. Much is left to the subjective evaluation of assessors. This could mean that errors could compound in the course of analysis and lead to implausible results.

Unit of analysis The OECD Guidelines do not provide a consistent, granular unit of analysis. Law, as manifested in the bodies of legal text, is the object of analysis of competition impact assessments. Previous competition impact assessments of the PCC (for example) have tended to focus on overall effects of the law, with special attention given to key provisions (while ignoring other provisions considered as merely procedural or boiler plate)Competition authorities may look at individual laws and analyze these for competition impact. However, a statute may not be the appropriate unit of analysis, since the competition impacts operate through key provisions that work with other critical provisions found in other laws. Looking at more atomic levels of distinct rules within provisions can also enable more detailed forms of analysis. To date, most of the PCC's published competition impact analyses analyze the law at the level of entire statutes, with some referencing key provisions, and nothing more granular.The competition impact analysis of the MNP Act only dealt with specific provisions of the law (and its implementing, administrative rules) without considering how its provisions could interact with each other, or with provisions from other laws.[34]

An entire law, although it can be described and interpreted as a unit, may be too cumbersome as a unit of analysis for competition impact assessment. A single law may contain multiple provisions, some of which may not be relevant to the analysis (like definitions and boilerplate). Even individual provisions will not be suitable, since a provision can still express several rules, each of which can be

[34]Olivo, Flores, and Bañez, "Competition Impact Assessment of the Mobile Number Portability Act (RA No. 11202)".

independently evaluated. Human evaluators with expert-level experience may be able to synthesize the effect of the law and placing it within an appropriate chain of causation and make a normative judgment. However, there is no unit of analysis with a standardized vocabulary or semantics that can be communicated, compared, and itself evaluated.An analysis methodology will need a way to isolate and represent individual rules.

Lack of proof Lack of provable, measurable basis for correlating textual provision with an anti-competitive effect. The purpose of impact assessment is to provide objective basis for decisions, resulting in policy that is better in the long te rm even if it clashes with immediate emotional considerations.[35]Even if a law is properly selected and studied at the appropriate level of description, the actual evaluation of competition impact is described on an intuitive, sometimes *ad hoc* basis. It is not encoded in a way that can be reliably communicated, proved, and further analyzed. For example: An evaluator conducting a competition analysis can warrant his;/her conclusions through a combination of legal and economic justifications, couched in some natural language. Another evaluator reviewing the assessment or a policymaker who wants to use the work, may not be able to step through the reasoning used in the analysis. Furthermore, across several assessments, we cannot be sure that there is consistency in the level of description or abstraction, the criteria for evaluation, as well as the modes of reasoning. We cannot even be sure that there is a consistent vocabulary used in the analysis.

The space for discourse for legal evaluation (arguments for or against a proposition) is potentially infinite. An evaluator may attribute, based simply on his own experience and intuition, all sorts of effects and consequences to the law being assessed. Without a rigorous framework, this could lead to mere proof by assertion, or affirmative conclusion from a negative premise.

For the field of competition law, the analysis is constrained by the tools of economic reasoning: i.e. a measure's effect on prices can provide a simple criteria for evaluating its effects. (Although

[35]Nakaizumi, *Impact Assessment for Developing Countries*, at 4.

there are many frameworks for the evaluation of effects and generation of alternatives, competition law's concern with markets makes it naturally aligned with economic thinking.) Economic thinking also practices the use of counterfactual analysis. Instead of speculating about the law in the abstract, we can construct a specific scenario where such a law exists, and model price effects under a given set of assumptions.

1.5 Overview of the Study Solution

1.5.1 Methodology

The problems with the canonical approach that are mentioned above relate to scale, rigor, consistency, and predictability. This can make the domain of competition impact assessment an appropriate setting for the application of computational law, which can look at legal rules as discrete units that can be evaluated through a consistent and transparent logical process. In this light, the question of competition impact can be structured as a computational problem.

This work aims to apply computational techniques to the following problems: 1. The representation of legal rules into discrete computable units; 2. The automated analysis and evaluation of legal rules for their possible competition impact. It hopes to introduce improvements to the problem of search and prioritization of laws: That is, the process of making an exhaustive mapping of concepts in a market in order to identify relevant laws, as well as applying the threshold tests in a consistent and rigorous manner. The critical initial step is to build a knowledge-base encoding the entities, attributes and relationships that are embedded in the law (this will be the semantic or ontological encoding). Parallel to this is the encoding of the rules (phrased as normative conditional statements) - this will be the normative encoding phase. Encoding also contemplates dealing with two sets of norms - 1. the norm that is being evaluated for competition impacts(the target norm); 2. the higher norm based on the OECD Guidelines (the reference norm) that can be used to evaluate any target norm.

In order to carry out automated reasoning of law, we have to encode legal norms into computational forms: Ontologies and description logic, will be used to define the descriptive component of the legal knowledge (in this case the OECD Competitive Impact Assessment tests) that we seek to encode. The analysis that can be performed on an ontology-based data structure can reveal implicit relationships between entities (such as inheritance,equivalence), as well as inconsistencies. However, ontologies only give us a part of the picture.These only provide static data about the semantics of entities and their interactions - but these do not reflect the legal constraints that act upon those objects, and how the semantics could be qualified, transformed, or annulled by such constraints. Besides the entities, their attributes and interactions - all these are subject to constraints and transformations based on law. Another way of putting it is that ontologies reflect only the "whats and the whos", not the oughts and ought nots that are contained in legal knowledge.[36]

After the ontology-based definition, the work shall cover the requirements of a logical system for representing important normative features of a body of rules: First, that it should capture the conditional nature of legal inferences; Second, it should involve modalities other than True or False - that is, it should work on normative states (for example, whether propositions are permitted, forbidden, or obligatory); Finally, it should also allow for the possibility of inferences being defeated by additional information. The chapter shall describe these features in turn, and propose Reified IO Logic as an encoding system that integrates all these requirements.

The choice of encoding system is based on Robaldo (2020)'s description of a computational knowledge base for legal rules,[37] which accomodates several levels of encoding:

1. **Legal text** - Written in a human readable language but tagged and structured through an XML-based markup (such as LegalDocML, an OASIS standard for legal markup). At this

[36]The extent to which ontologies can model the semantics of entities and events is elaborated in Chapter 5. The limitations of ontologies in expressing the normative knowledge contained in law is explained in Chapter 6.

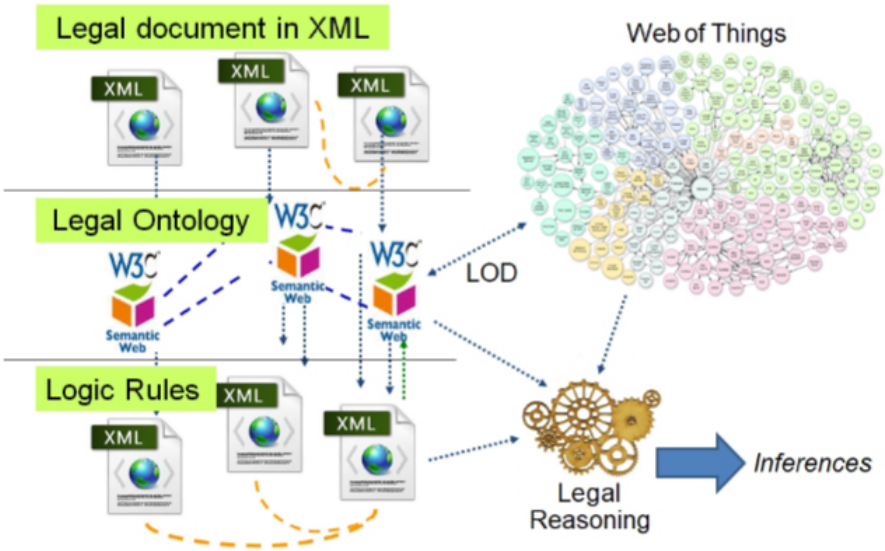
[37]Livio Robaldo, Cesare Bartolini, and Gabriele Lenzini. "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML". in: *Proceedings of the 12th Conference on Language Resources and Evaluation (LREC 2020)*. Marseille, May 11–16, 2020, pp. 5688–5697, at 5688-5689.

level, the system designer encodes the law as is, but provides markup for some sections in order to signal the structure of the document, as well as highlight concepts that are relevant to the ontology and logic layers. This allows systems to associate these elements with subsequent logical encodings the represent their meaning. This will allow components of the text to be linked to the subsequent encodings and support automated processing.

2. **Legal ontology** - This consists of the formalized naming and definitions of concepts that are contained in the human-readable rules, as described in the previous chapter. Concepts and relationships are encoded in OWL, and will serve as the predicates to be used in Thehe normative logic layer. The ontology can also be described in terms of Description Logic, and can support some analysis. However, this ontology layer alone is not fit for legal reasoning, as it does not account for deontic aspects of the rules, or accounts for their defeasibility.

3. **Normative logic** - This layer represents the normative content of the rules, represented as logical formulae. This logic layer is formalized in a defeasible form of deontic logic and then encoded in LegalRuleML.

Figure 1.1: Layered Architecture of Encoding(Robaldo, 2020)



While the current section (Chapter 1) provides an overview of the motivating problem for the work, the rest of the dissertation will proceed along the following lines:

- Chapter 2 - Provides an overview of computational law in general, and its application to the field of competition law;
- Chapter 3 - Mounts a defense of the use of the logical theories underlying the computational approach used, justifying its use for competition impact analysis;
- Chapter 4 - Describes in detail the domain to be encoded - a section of the OECD guidelines on competition assessments, with a particular focus on the primary subjects of Section A - exclusive rights and barriers to entry;
- Chapter 5 - Provides an overview of the methodology for encoding the semantic content of the above domain - from Description Logic to the Object Web Language. It will then describe the domain as encoded;
- Chapter 7 - Illustrates the applications of the encoding to facilitate evaluation in threshold testing;
- Chapter 8 - Discusses the limitations and normative/ethical concerns with using computational law (and AI in general for the task of legal reasoning);
- Chapter 9 - Provides conclusions and directions for further study.

1.5.2 Contributions and Originality

Although the use of computation to tackle legal problems in general (and competition law problems in particular) is not new[38] At time of writing, this work is unique in its use of symbolic computational techniques for the competition impact analysis - This is the first work to use the logical framework for the knowledge of competition law norms. This is also the first such work that

[38] See further discussion in Chapter 2 of this work.

applies symbolic computational techniques of compliance checking to evaluation of legal knowledge based on competition norms. This work can also be distinguished from the literature through its conception of individual rules (modeled as if-then statements) as the granular level of encoding and analysis.

Proof of concept for encoding Before the computational analysis can be applied, however, it is necessary to establish whether or not (or the degree to which) the domain of competition law can be encoded. This study is concerned with an initial exploration of the computational representation of problems related to the canonical approach, based on the OECD guidelines and related materials. The goal of the dissertation is to provide a proof of concept for the encoding and analysis of competition law norms. Specifically: 1. Design and encode an ontology of concepts in the OECD guidance rules 2. Design and encode representation of the normative content of the OECD guidance. This will test just how much competition law is susceptible to a computational approach - e.g. what parts can be encoded, what design and technology choices will make the approach feasible.

This component of the work contributes to the abstract aspect of the computational law discussion, particularly the question of whether or not it is possible to have an accurate encoding of the law into a formalism, and if so - whether this is equally applicable for all fields of law, or if some laws are more likely to be computable than others.

Theory testing Although there is a lot of theoretical work for the encoding and analysis of norms, as computational structures, there is a relative dearth in practical applications. The study is thus also concerned with theory testing, determining the "fit" of proposed approaches to logical encoding (such as Descriptive Logic for semantics, and Reified I/O Logic for norms) as applied to a hybridized domain of competition law, which will involve both legal and economic reasoning. Model-building (and testing) is important at this stage of development of the field -in order to test, validate, critique theories and methods (which are varied). And to explore both advantages and limitations of the computational approach to different domains of legal reasoning. In the course of the encoding

process, the work will document both the advantages and limitations of the formalization approach (e.g. what details from the underlying legal texts are captured and lost).

Tool testing Finally, the study engages in tool testing - the technologies covered in this work such as the Web Object Language(OWL), the Shapes Constraints Language(SHACL), and foundational ontologies such as Finance Business Ontology(FIBO) have not yet been used to encode and analyze domains of competition law or competition impact assessment. The work also contributes to the semantic web project of building representations of various knowledge domains.[39] Although there are many extant semantic web constructs (ontologies, queries and shapes to define constraints) available online, there are few such constructs specifically related to law, and none relating to competition.

Computational law and good institutional design Nakaizumi (2022) suggests that impact analysis will be more effective if embedded within good institutional design with the following components: 1. Established ground rules; 2.Guidelines on the conduct of assessments;3. Separate institutions tasked to conduct assessments;4. Involvement of economists or other scientific researchers. A computational approach to competition impact assessment directly contributes to the first two components by making explicit the standards, definitions, and procedures of an assessment exercise. To the extent that the computational encoding is transparent, portable, and accommodates the input of experts, it can also contribute to the last two requirements.

Unit of analysis The threshold test of competition impact assessment can be stated more formally as follows: Given a set of rules (i.e., the rules that cover an industry) - does it comply with or diverge from the idealized norm of the threshold test? In previous assessment exercises, to make things manageable logistically, the author has proposed making individual provisions the unit of analysis. However, even a provision can still express several rules, each of which can be independently evaluated. Therefore rules will serve as our unit of analysis.

[39] More on the semantic web, its relationship to computational law and its applications for this work in Chapter 2

1.5.3 Overview of Methodology

This work puts forward computational approaches to the problem of determining relevance as well as making a threshold evaluation of a norm.

The proposed work is based on restating the threshold testing phase of competition impact analysis in computational terms:Given a rule within a relevant law, is the rule **compliant** with the norms laid out by the threshold test? Would it be possible to express competition norms in terms of logical constraints, and then check whether a particular instance of a norm complies with such constraints?

From a computational point of view, the problem of competition impact assessment is a problem of logical comparison and evaluation. It involves comparing the provisions of the law that cover a sector with a set of standards, and then evaluating the extent to which the law complies with the standards. The standards can refer to the OECD threshold tests (and are further elaborated in the economics literature, usually based on models of a competitive market). In order to proceed with automating this evaluation, a computational law system will require: 1. A system for encoding the content of legal text, as well as 2. Algorithms that can process these encodings.Table 1.1 presents possible approaches to both encoding and the subsequent analysis. Note that this work will only explore a subset of these approaches (e.g. as will be discussed at length below, the work will forego dealing with the relevance problem).

Computational law requires us to turn norms into discrete units of analysis. Entities and relationships described in the law can be turned into predications in Descriptive Logic before being programmed into the Object Web Language (OWL). In the case of the normative content of the law, each provision in a legislation can be further atomized into: normative conditional (if-then) statements that can then be programmed into the Shapes Constraint Language (SHACL).

Table 1.1: Encoding and Analysis Approaches

Problem	**Encoding**	**Analysis**
Relevance Testing: Does the law map with the industry being assessed? (Actors, transactions)	Ontologies (Ontology Web Language)	Reasoning engines to determine relationships: - No mapping? - Identity? - Classification? - Mereological? - Inference?
Threshold Testing: Given a specific rule within a relevant law - How does this rule relate to the norm of the threshold test?	Inference rules (Prakken, Sartor) - LegalRuleML	Argumentation Frameworks Propositional networks

Part of competition impact analysis is concerned with generating regulatory alternatives that can fulfill policy goals with less anti-competitive impact.[40] The final chapters of this work will explore computational approaches to generating these alternative norms.

1.6 Scoping and Limitations

Constraining the problem The goal of the proposed work is to use the computational approach for competition impact analysis. Particularly, for the problems of relevance and threshold testing. The scope of the study is further limited to applications for the special case of the Philippine digital payments sector - where both the assumptions, definitions, and constraints are more explicit. It is asserted that the digital payments sector is more mappable into computational terms due to the following factors: 1. Digitalization, through the act of requirements specification and programming, constrains the semantics of terms. That is, specific terms and transactions need not correspond to open-textured meanings but to specific software objects or routines; 2. The digital payments sector, due to the nature of its subject matter and risks, has evolved out of necessity to be more specific

[40] OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*.

and rule-driven regarding its norms.[41] Since payment and settlement rules have to be embedded in software to facilitate large volumes of automated transactions, digital payment rules will often require specific criteria (e.g. concrete KYC requirements, or transaction amount threshold instead of open-ended standards).

The study can be limited to well-defined standards in competition - i.e. those that are already extensively documented and tested in the economics literature, and so can be a source of explicit rules on how a competitive market ought to behave.The CIA process, in evaluating laws, reflect some higher norm on how a market is supposed to behave (in this case, legal and economic norms as codified in the OECD Guidelines), and that the law should enable, or not stop that preferred standard of functioning. As to the laws that will be evaluated, the plan is to focus on a segment that is already digitized and subject to very specific constraints. The digital payments sector is a good candidate. Out of necessity, the sector does not involve many entities and transactions with open-ended states. It is also a field characterized by extensive, semantically rich constraints from industry standards, government regulations, user contracts, and the functionality of the digital platforms themselves.

Subset of competition law problems The literature on computational anti-trust[42] suggests that computational techniques for modeling and analysis can be applied to a wide set of competition law problems, from enforcement against unilateral actions (such as abuse of dominance), to the analysis of mergers for potential anti-competition effects. There is no current discussion, however, as for applying computational techniques to the problem of competition impact analysis. This works attempts to fill that gap.This study will encode competition law concepts and norms, and can have applications for a range of competition law problems, including enforcement against the unilateral acts of private economic actors, as well as merger reviews. One major difference between these problems (enforcement, merger analysis) and the competition impact analysis problem is that the former involves the conduct of private individuals and their interactions in a market.

[41] Please see discussion in Chapter 3 regarding the suitability of the computational approach to specific types of laws.
[42] See further discussion in Chapter 2

On the other hand, the competition impact analysis problem is ostensibly concerned only with government conduct, as reflected in the text of its laws and regulations. Although the modeling and analysis techniques/approaches discussed can be applied to this set of problems (enforcement, merger review) our focus will be confined to the possible effects of law and related conduct on the state of competition itself. For this work, however, the scope is limited to applications for competition impact assessment. This means that the concepts and norms to be encoded will be focused on public behavior, i.e. the conduct of government entities, as manifested through legal and regulatory text.[43]

The following scenario represents the kind of use cases, representing the typical level of analysis that the work contemplates to enable: The processing of relatively simple normative questions at scale. The University of the Philippines Law Center,[44] based on its statutory duty of assisting with law reforms, has participated in what amounts to be general impact assessments of both existing laws and pending legislation. While it endeavors to provide in-depth analysis for many key legislation , oftentimes what is required is a very broad view of a proposal's impact across an existing regulatory background. This might be the case when the government is contemplating accession to a comprehensive treaty (such as the Comprehensive and Progressive Agreement for Trans-Pacific Partnership or CPTPP), and there is a need to determine what internal rules will need to be changed, or what vested rights may be affected, by the accession. The analysis required to resolve this problem does not need to delve into both current and historical meaning of the law, as well as its consequences for a whole range of policy concerns - It simply requires a determination as to whether or not an internal rule is compatible with the requirements of the treaty.

[43] Nevertheless, the problem of competition impact analysis has the same shape as other problems that involve the involve analyzing and evaluating an extensive body of rules (such as: 1. gap analysis; 2. impact analysis; 3. compliance analysis). They all involve some form of legal comparison and evaluation - old law against new law, n-level law versus n-1 level law, etc. So an advance in the solution of one problem can contribute to the other.

[44] The UP Law Center is an organization with its own legal charter Republic Act 3870, under which it is tasked, among others, to "to undertake legal studies and researches on request from the various agencies of the government concerned with law reform, including the committees on Revision of Laws of the House of Representatives and the Senate". Congress. *Charter of the U.P. Law Center.* June 12, 1964.

Limited to the OECD Guidelines The legal and economic literature provides a rich source of possible definitions, as well as norms that can be encoded. However, this work limits its scope to the text of the OECD Guidelines for Competition Impact Assessment.[45]

As of time of writing, the OECD appears to have the most substantially documented standard in the public record - with three volumes worth of background and operational guidance. The guidelines from APEC is not as documented. Similarly, the guidelines from the World Bank is not open to the public. The OECD Guidelines are documented not just as to competition analysis norms, but also related concepts that can provide further granularity to our computational encoding. These guidelines also already takes into account the accepted economic literature, and largely eschews abstract philosophical notions and adopts practical tests based on actual cases.

Whenever appropriate, the work will also refer to other works of the OECD to provide further definition to the encoding. Examples of such references includes the OECD's work on unilateral acts (for elaboration of market power) as well as its studies on barriers to entry.[46]

Limited to the threshold analysis problem Most legal problems are concerned with trying to determine what set of rules is even applicable in the first place. This is not the usual domain of computational law.[47]Instead, it is confined to mechanical application of rules so deemed applicable.The encoding and analysis assumes that the human evaluator has selected both the sector and its relevant laws - the study is not concerned with the relevance problem. As previously discussed, the definitions of the sector (and the subsequent but related problem of selecting relevant

[45]The OECD provides a "Competition Assessment Toolkit", proposing for competition agencies a set of guidelines for the conduct of competition impact assessments. The guidelines are published in three volumesVolume 1 (Principles), Volume 3 (Guidance), and Volume 3 (Operations Manual) OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*; OECD. *Competition Assessment Toolkit - Volume 2 (Guidance)*. 2019. URL: https://www.oecd.org/daf/competition/45544507.pdf (visited on 10/10/2023); OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*.

[46]See Organisation for Economic Co-operation and Development. "Barriers to Entry: Key Findings, Summary and Notes". In: Roundtables on Barriers to Entry. Vol. 58. OECD Competition Law and Policy Working Papers. Mar. 6, 2006. DOI: 10.1787/8bb30107-en. URL: https://www.oecd.org/en/publications/barriers-to-entry_8bb30107-en.html (visited on 01/18/2026).

[47]Robert Kowalski and Marek Sergot. "The Use of Logical Models in Legal Problem Solving". In: *Ratio Juris* 3.2 (July 1990), pp. 201–218.

laws) is tied not just to describable criteria (for markets and participants) but determined as a matter of policy or convenience by the evaluator, or other stakeholders to the competition environment (the competition authority, executive departments, or the legislative). Determining whether or not a law is relevant to the sector being studied is an important part of competition impact analysis. It is an interesting problem that could benefit from a computational approach: However, it is a problem too broad for the strictly normative-logical approach adopted by this work. The problem would involve a whole other set of theories and tools.[48]

Limited encoding and deployment The work is not meant to be an exhaustive encoding of all possible ontological definitions of concepts in the OECD tests as well as how these concepts are used in rules. The intent is to provide enough examples to illustrate the feasibility and advantage of the computational approach. Neither does the study aim to provide a complete encoding of all relevant entities, relationships, and norms for competition. Instead, it will focus on demonstrating the feasibility of encoding those entities and norms related to the conduct of public authorities.

As to the programming aspect of the project: Although the work will feature source code to demonstrate the proposed encoding approach, it is not the goal of this work to create a full application that can automate the competition impact assessment process, even for the limited domain discussed in Chapter 4. While the work will not result in an exhaustive, application-ready encoding, it provides an opportunity to document design choices, constraints, and other considerations that can improve future computational approaches to the subject.

1.6.1 Limited Scale and Efficiency

As an initial, exploratory approach to the problem of setting competition impact analysis within computational law - the scale of the encoding and analysis is modest - the coverage of the encoding applies only to the first subsection of the OECD guidelines, and only several provisions of the

[48] See for example Walid S Saba. "Reinterpreting 'the Company a Word Keeps': Towards Explainable and Ontologically Grounded Language Models". Preprint. May 2, 2024. URL: https://arxiv.org/abs/2406.06610.

target norm were encoded. Even with this limited scope, the encoding stage was a difficult, time-consuming process. Encoding had to be adjusted and reworked often to balance not just abstract considerations (such as isomorphism with the domain being modeled), but also limitations of the encoding technologies.

Given the above experience, the objection may be made that the computational approach based on symbolic logic may be too inefficient to realize at scale and may not be "worth the candle", i.e. that the costs associated with encoding and analysis cannot make it a viable alternative to either manual analysis, or analysis through machine learning. There is a temptation to simply dismiss cost and efficiency concerns as outside the scope of purely academic research and development, or that further study may be able to address these concerns in the future. However, efficiency problems are critical to the adoption (or non-adoption) of a technology, and in this case it may be naive to rely on the open-ended growth of the underlying technologies. Thus, we shall briefly touch upon some aspects of the cost/efficiency problem and some possible solutions.

Maintennance and co-drafting The encoding cost can extend not just to the initial encoding of the law, but the need to keep such encoding up to date, as the law is amended or superceded by other laws. As the code base of encoded laws expand and mature, there could be an escalation of costs associated with re-testing and debugging it with each subsequent legal reform.

The encoding and maintenance process can be made more cost efficient by adopting a co-drafting regime - where the statute and its "digital twin" - a machine readable version of the norms - are written concurrently. The co-drafting exercise will have the additional benefit of forcing legislators to clarify the semantics of the law early on, and test the law for possible logical contradictions, ambiguities, and loopholes, prior to passage of the law.

Talent pool requirement Encoding into computational forms was not something that could be grasped intuitively - It required a baseline knowledge of at least two semantic web languages (OWL

and SHACL), as well as their underlying logical accounts. It required from the author not just familiarity with the technologies, but also the domains to be encoded (law, economics).

In future applications, the requirement of specialized, highly trained talent who can link technology with these domains may result in a translation bottleneck that can prevent enough laws from being encoded to make analysis viable.

The bottleneck can be addressed by a training regime for legal professionals that introduce computational concepts - and how legal texts can be encoded. As generations of "digital natives" come of age and join the legal profession, it may become easier for both law schools and continuing legal education to bridge the law-technology divide. Similarly, the talent pool for encoding can be expanded by removing the burden from highly priced professional lawyers, and onto new allied legal professions (i.e. "legal technologists") with specialized technical training.

Amortization of costs As mentioned earlier, the encoding process involves a significant setup cost. However, these costs may be amortized across greater number of units encoded. The costing model for this approach to computational law is not well studied. However, assuming the conventional cost curve for technological development applies, then we can expect marginal costs to go down with every additional unit (rule) encoded.

Thus, the experience gained from the initial encoding(research and development costs) can be amortized through every additional rule encoded, and the overall costs can be managed through efficiencies of scale.

Focusing purely on the upfront costs can make us lose sight of the greater long term return investment while the initial encoding cost (even considering lower marginal costs for every new rule), the analysis costs for high volume transactions, coupled with benefits such as lower bureaucratic overhead, fewer errors, faster processing, can more than justify the costs.

1.7 Miscellaneous Considerations

Automated reasoning can also allow competition authorities and policy makers to make extensive evaluations efficiently and at scale. A responsive competition assessment system should not only evaluate retroactively, for existing laws, but also conduct the exercise for new or proposed legislation. While it is possible for each new law to carry not only prospective effects in a particular subject matter - it is also possible for it to interact with the existing legal environment in a way that would change the competition impact of prior laws. A new baseline understanding of the competition impact of the entire legal environment may have to be inspected with the passage of each new law, compounding the complexity of the task and adding to the burden of competition authorities. Part of this regression analysis can be automated through the computational approach.

It should be noted that the primary concern of this paper is evaluating specific rules for competition effects into a computable problem.This is achieved by developing workflows and toolkits to demonstrate the feasibility of a computational approach to competition law - although not necessarily a mature application ready for frontline use. The work will explore formalizations for representing the above problems in a way that can be processed by computers. The emphasis is in developing tools for the facilitation of competition impact assessment. This assumes that an appropriate body is responsible for conducting such assessment. This and other features of a competition assessment regime, such as the location of the assessment in the larger policy development process, the involvement of the competition agency, will not be within the scope of this work. Although Computational Law is usually associated with automation of legal determinations though a computer - it is not the goal of this work to implement an automated counterpart for the sections of the competition impact assessment process that will be encoded into a computational form. Some experimental code might be featured in order to demonstrate the feasibility of some proposals, but these are not production-quality implementations. It should be noted that the advantage of the computational approach goes beyond machine execution of routine legal tasks, but in helping develop notations through which we can understand and share problems of legal reasoning.

Why is all the effort towards abstraction preferable to the usual intuitive approach? In addition to giving us scale and automation, the computational approach can help us in two ways: 1. It can help us make our analysis more rigorous, and 2. It can help us make our analysis more transparent. The process becomes more rigorous by eliminating ambiguity and unstated assumptions. Instead of intuitive guesses, the computational approach makes available firm conclusions derived from explicitly defined premises, using accepted rules of inference. The process becomes more transparent because the premises, the chain of reasoning, as well as the conclusion can be represented and made available for inspection and correction. Sharing legal knowledge through a formalized notation can help us build richer systems of legal knowledge.

It should be noted that regardless of the formalization, inference rules, defeasible deontic logic is already embedded in the practice.[49] By analogy, just as it is possible to prepare meals without understanding chemistry or nutrition, legal actors can render determinations even without familiarity with logical formalisms. Nevertheless, just as some foundational knowledge of chemical processes, nutritional properties can optimize food preparation, understanding logic can enhance the precision and efficacy of legal reasoning.

[49] See discussion on Chapter 3 on how logic is embedded in practice. Irrespective of the specific formalization, forms of logic (deductive, inductive, defeasible) are embedded in the practice of law. It is possible to conduct law as a practical matter without appeal to abstract reasoning.

Chapter 2

An Overview of Computational Law

2.1 Historical Background

The project of applying computational techniques to the legal domain - e.g. encoding law into computational terms, and mechanically applying or analyzing these - was among the earliest directions of artificial intelligence research. Despite its early promise, however, the approach did not bear fruit.[1] During the 1980's there was initial optimism about the prospect of computers performing automated legal reasoning. Grossman summarizes the research and programming activity towards this end. They note that while computers cannot replace lawyers, these machines can, in time nevertheless run "legal reasoning systems" that can assist attorneys.[2] Computerized legal reasoning offered speed, reliability, and the ability to carry out numerous, repetitive tasks. It could also provide a consistent application of the law.[3] It was also hoped that the availability of such systems could have knock-on effects on legal reasoning itself, molding the thought processes of legal professionals

[1] See Michael Genesereth and Nathaniel Love. "Computational Law". In: *Proceedings of the 10th International Conference on Artificial Intelligence and Law - ICAIL '05*. The 10th International Conference. Bologna, Italy: ACM Press, 2005. ISBN: 978-1-59593-081-1. DOI: 10.1145/1165485.1165517. URL: http://portal.acm.org/citation.cfm?doid=1165485.1165517 (visited on 09/18/2021), at 205.

[2] Garry S Grossman and Lewis D Solomon. "Computers and Legal Reasoning". In: *ABA Journal* 69 (1983), pp. 66–70, at 66: "Primarily, a legal reasoning system would serve as a repository of knowledge, outlining the general parameters of the law. In lieu of searching through a treatise or similar task, given a specific factual situation, the system could be relied on to present only the relevant law."

[3] Ibid.

towards logical rigor, and force the field to be more explicit about its assumptions.[4]

2.1.1 Initial Approaches

Early attempts at replicating legal reasoning through software tried to emulate the fact that lawyers employed both deductive and analogical reasoning when working on a case:[5]

1. **Deductive Approaches -** These involve looking at conditions, propositions in the law as well as fact patterns, and then making inferences towards legal conclusions. For example: "If *A* or *B* then *C*". The computer stores representations of operations (e.g. the inference from *A* to *B*), as well as their premises (e.g. what *A*, *B*, and *C* stand for).

2. **Inductive Approaches -** These systems sought to model the way lawyers actually worked, i.e. reasoning from specific precedents to a new specific case (i.e. also know as "case-based reasoning").

3. **Analogical Approaches -** Means taking into account analogous cases, i.e., those that may have different fact patterns but similar relationships. A computer can attempt to reason by analogy by searching for relationships in fact patterns similar to those of the case at hand. The system can apply the rule of one case to another based on their similarity.

The deductive approach is best represented by the so-called "rules based" reasoning systems. These operated by applying fixed statutory rules, in a top-down manner, into specific cases. One example of the deductive approach was JUDITH, developed in the early 1970's by Walter Papp and Bernhard Schlink. The law was modeled as a set of premises (as defined by its programmers). The user goes through the premises that may be stereotypical for a given problem, determining whether they applied or not (True or False). Based on this knowledge, the system attempted to determine whether a cause of action exists under a given set of facts.[6]

[4] Grossman and Solomon, "Computers and Legal Reasoning".

[5] Ibid., at 67.

[6] Another example of a system from this period using the deductive approach is Hellawell's tax planning systems - one used to determine the treatment of redemptions, and another for the optimal choice of foreign subsidiary. Both

One of the more famous systems using the inductive approach is the CATO system, which covered the domain of the Trade Secrets Act. It used "deconstructing factual-extracting elements-comparison method" to conduct analogical reasoning conclusions of pending cases, through comparison with selected similar cases.[7]

On the other hand, the TAXMAN system by McCarthy had an approach similar to analogy.[8] Instead of asking specific questions (like Helawell's system) it maintained an internal representation of: 1. The fact pattern at hand and 2. Fact patterns that are inherent or usually associated with corporate reorganizations. These representations come in the form of "semantic networks", compound statements elaborating the legal relationships in these fact patterns. Users were expected to enter a fact pattern (in a formal, structured language). The computer would then search through the semantic network for similar relationships. This required all relevant relationships to be thought of beforehand and represented in a formal language.[9]

The systems mentioned in the previous paragraphs were limited in terms of the legal problems they addressed. They dealt with areas where there were fewer ambiguities, or had rules that are susceptible to mechanistic analysis. They were also limited by the technology available at that time. Making the internal representations of facts and laws involved complexity and a lot of resources. None of the programs had standardized, user-friendly interfaces. Even then, their solutions were often superficial and were thus of limited value in real world settings.[10]

systems involved no attempt to create an internal model of the relevant laws. Instead, the explicit tests were programmed directly into the system, and tailored to specific problems. The design and implementation of both systems were not adaptable to other areas of law ibid., at 67.

[7] Caixia Zou, Fangchun Li, and Yi'nan Dong. "The Rationale and Approach of the Legal Expert System Construction". In: *Beijing Law Review* 13.01 (2022), pp. 204–216. ISSN: 2159-4627, 2159-4635. DOI: 10.4236/blr.2022.131013. URL: https://www.scirp.org/journal/doi.aspx?doi=10.4236/blr.2022.131013 (visited on 11/25/2025), at 205; Kevin D. Ashley. "Designing Electronic Casebooks That Talk Back: The CATO Program". In: *Jurimetrics* 40.3 (2000). URL: https://ssrn.com/abstract=4082451.

[8] Grossman and Solomon, "Computers and Legal Reasoning", at 67-68.

[9] On the other hand, Meldman's query-based system for assault and battery cases is cited as another example of a system that applies analogy. It works by taking in as input a series of word groups that describe the facts of a case. The system will then state whether it can identify cases with similar fact patterns. It can be classified as a research tool rather than a system for automating legal reasoning. ibid., at 69.

[10] Ibid., at 69.

2.1.2 Early Efforts at Formalized Models

Kowalski's work represents early attempts at building logical models for law. "Model" implies a degree of abstraction of the problem domain, capturing only certain features deemed essential to the law (and consequently, ignoring others).[11] These early efforts employed Extended Horn Clauses (A if B and C, where the conclusion A expresses an atomic relationship and the conditions $B \ldots C$ are atomic relationships or their negations). For example a clause of the British Nationality Act of 1981 which states "A person born in the United Kingdom after commencement shall be a British citizen if at the time of birth his father or mother is...a. a British citizen;"[12] can be formalized into the following set of Extended Horn Clauses:

1. x becomes a British citizen by section 1.1a.
2. if x was born in the United Kingdomn on date y
   (a) and date y is on or after commencement;
   (b) and x has a parent z;
   (c) and z is a British citizen on date y

From this foundation, Sergot later outlines a project to encode the British Nationality Act into logic through the PROLOG language - to enable mechanical determination of the act's consequences.[13]

[11] Kowalski and Sergot, "The Use of Logical Models in Legal Problem Solving", at 202.
[12] Ibid., at 203.
[13] M. J. Sergot et al. "The British Nationality Act as a Logic Program". In: *Communications of the ACM* 29.5 (May 1986), pp. 370–386. ISSN: 0001-0782, 1557-7317. DOI: 10.1145/5689.5920. URL: https://dl.acm.org/doi/10.1145/5689.5920 (visited on 10/27/2023), at 370.

2.2 Definition and Contemporary Applications

2.2.1 Definition of Computational Law

According to Genesereth:“Computational law is that branch of *legal informatics*[14] concerned with codification of regulations in precise computable form."[15] A similar subfield is what Sartor et al calls “Computational models of the law”, which is concerned primarily with “the definition of computable models for legal knowledge”.[16] Sartor notes than this modelling can occur at several levels: From modelling law at the surface level as documents or data (the approach taken by legal search engines), to looking at ever more granular components of legal reasoning, such as modelling legal norms, legal concepts, cases, and legal interactions. While some of these technologies are at the theoretical research phase, others are deployed of more extensive legal /compliance support systems enhancing workflows for electronic data discovery, argument creation, prediction, etc.[17]

In terms of practical applications - it can provide the basis for computer systems performing compliance checks, legal planning, the analysis of regulations, and related functions.[18] Many computer applications aid lawyers in their tasks, but these are not within the ambit of the term. Examples include legal databases to find the law, and office productivity suites to help the practitioner prepare briefs, or systems to automate the backroom functions of the law office. In these instances,

[14]“Legal informatics is defined as the study of information, its technology, and it implication and impact in the field of law”. This is to be differentiated from “Computer law”, which is concerned with “problems relating to the social implications of information technology in the field of law.” Christopher L. Hinson. “Legal Informatics: Opportunities for Information Science”. In: *Journal of Education for Library and Information Science* 46.2 (2005), p. 134. ISSN: 07485786. DOI: 10.2307/40323866. JSTOR: 10.2307/40323866. URL: https://www.jstor.org/stable/10.2307/40323866?origin=crossref (visited on 11/02/2023), at 134-135.

[15]See Michael Genesereth. *Computational Law: The Cop in the Back Seat*. CodeX: The Center for Legal Informatics Stanford University. 2015. URL: https://law.stanford.edu/publications/computational-law-the-cop-in-the-backseat/ (visited on 09/18/2021), at 2.

[16]Giovanni Sartor et al. “Computable Models of the Law and ICT: State of the Art and Trends in European Research”. In: *Computable Models of the Law and ICT: State of the Art and Trends in European Research*. Lecture Notes in Artificiai Intelligence. Germany: Springer-Verlag, 2008, at 5-6.

[17]Ibid., at 6-8.

[18]See Michael Genesereth. *What Is Computational Law?* Stanford Law School. Mar. 10, 2021. URL: https://law.stanford.edu/2021/03/10/what-is-computational-law/ (visited on 04/19/2026), Computational law has many potential applications. However, its primary focus is in compliance management - the development and deployment of computer systems capable of assessing, facilitating, or enforcing compliance with rules and regulations.

the legal reasoning is still performed by the human agent. The computer performs symbolic analysis for purposes of retrieval and presentation of data, without any recognition of the rules as such.[19] From a pragmatic perspective, Computational Law is important as the basis for computer systems capable of doing legal calculations, such as compliance checking, legal planning, regulatory analysis, and so forth".[20] The touchstone of the computational project is the creation of "Codex Machine" which contains within itself an extensive databases of encoded rules, and with all the required computational resources, provide responses indistinguishable from that made by a legal professional.[21] Despite the recency of the term, its goal is shared by early projects in artificial intelligence, which saw the legal domain as a natural site for the application of computational techniques.[22]

The prevalent approach to meeting these goals has two components: 1. First, representing law (and surrounding facts) into a formal logical form and 2. Second, the ability to process those representations to assist in legal determinations. This means: That it would be possible through computational techniques, to arrive at consistent, correct (or at least plausible) legal conclusions from given set of premises and operations.

[19] See Genesereth, *Computational Law: The Cop in the Back Seat*, at 2-3 for a proposed example. According to Genesereth the Turbo Tax program is a computational law application. The user supplies values, and the program makes computations of the user's tax obligation. When prompted, it can explain its results by making references to the applicable tax law. Legal rules (whether or not taxable, the base, rate, and tax due) are encoded (however indirectly) as code, and the result of the processing is a legal determination - whether or not tax is due, and how much. But see Hans Andersson. "Computational Law: Law That Works Like Software". CodeX – The Stanford Center for Legal Informatics, Feb. 10, 2014. URL: https://www.academia.edu/9286857/Computational_Law_Anderrson_and_Lee, at 3-4. There is a tendency to invoke a Turing test analogue for computational law system: "Any system whose users inputting, through whatever interface such system might present, a legal query to obtain a legal response would find themselves unable, given only the response, to determine whether a legal professional...had provided the system's response." Andersson rejects this criteria because it would include systems that only outwardly appear to be computational law without actually solving its fundamental problems. Based on his rejection of the Turing or "imitation" principle of what constitutes computational law, Andersson argues that Love and Genesereth's inclusion of Turbo Tax within the definition is inaccurate. Although the program appears to replicate the behavior of a tax professional - it does not formally represent laws, or performs automated reasoning based on those representations. This author notes that the whole point of the imitation principle is that the intent of representation does not matter - thus avoiding many philosophical questions.

[20] Genesereth, *Computational Law: The Cop in the Back Seat*, at 2.

[21] Andersson, "Computational Law: Law That Works Like Software", at 16.

[22] Genesereth and Love, "Computational Law", at 205.

These determinations can be descriptive, recreating in computational form the law as it is, and guiding its users in evaluating whether certain actions or states of the world are in accordance with the encoded rules. It can also be prescriptive, meaning the rules as encoded can be analyzed and evaluated against standards (such as efficiency), or their alignment with other rules, in order to arrive at more suitable rules.[23] Despite its aspiration of being a visible system of explicit rules, so much of the law is actually dependent on tacit knowledge, i.e.: Other, higher order rules (for determining applicability, validity, interpretation) entities and concepts that are not provided in the legal text. Part of the project of computational law is to surface that tacit knowledge.

2.2.2 Recent Applications

Alarie (2016) provides a hypothetical application in tax law. Once legal standards and factual situations are encoded into computational form, it will be possible to query whether or not a person is an independent contractor or an employee. Although there may be no bright line rules, there is a standard called the total relationship test that looks at various aspects of the relationship, including: intention, exercise of control, ownership of tools, worker's risk of loss, and chance of profit, integration into the hirer's business activities.A machine learning algorithm can reduce the hundreds of cases that use the standard into a questionnaire that helps us arrive at the correct conclusion 98% of the time.[24]

Driven by evolutionary pressure (competition with firms with access to computational tools for more sophisticated tax avoidance), governments will adopt computational systems to close the loopholes and approach a complete specification of tax law.[25]Furthermore, as machine learning involves optimization amidst multiple dimensions of preferences and constraints. Applying this to tax law - It may be possible for an algorithm to find an efficient equilibrium that optimizes policy

[23]Andersson, "Computational Law: Law That Works Like Software", at 7.

[24]Benjamin Alarie. "The Path of the Law: Toward Legal Singularity". In: *University of Toronto* 66.4 (2016), pp. 443–445. ISSN: 1556-5068. DOI: \https://doi.org/10.3138/UTLJ.4008, at 5.

[25]Ibid., at 8-9.

tradeoffs.[26]

Example computational law application is Fides, a language/platform that allows the automation of data privacy compliance. It works by mapping data objects into use constraints (by encoding data privacy policies into a programming language).[27]

Computational law is bound to emerge and disrupt given: the economies at stake, as well as the inertia of the incumbents. Will start with specialized systems, later on converging into general purpose ones. Will require first - harmonization, and later the unification of legal text. Andersson calls for an emphasis on actually building the systems, the applications instead of piling on more theory.[28]

2.2.3 Computational Antitrust

Schrepel acknowledges that computational tools are in fact widely deployed in the legal field (employing data mining, machine learning, etc.)[29] Computational antitrust is a subfield of computational law - the use of computational methods to improve antitrust agencies' ability to detect analyze and remedy anti-competitive practices, manage merger control, better design and monitoring of antitrust policies. Computational antitrust is a subfield of computational law - the use of computational methods to improve antitrust agencies' ability to detect analyze and remedy anti-competitive practices, manage merger control, better design and monitoring of antitrust policies.[30]

Computational antitrust is a project (led academically by the Stanford Law School) which aims to address growing complexity enforcement through the use of informatics - for: 1. Automation of procedures - Involves streamlining the interaction between agencies and market players.[31] 2.

[26] Alarie, "The Path of the Law", at 10.

[27] Ethyca, Inc. *What Is Fides? - Fides*. Nov. 25, 2024. URL: https://ethyca.com/docs/ (visited on 01/15/2023).

[28] Andersson, "Computational Law: Law That Works Like Software", at 24.

[29] Thibault Schrepel and Teodora Groza. *Computational Antitrust Worldwide: Fourth Cross-Agency Report*. June 18, 2025. URL: https://ssrn.com/abstract=5305055, at 3.

[30] Ibid., at 4.

[31] This includes: automated merger filings. real-time compliance checks, digital discovery

Enhancement of analysis - Using AI to detect patterns humans cannot see, effectively closing the gap between diagnosis (identifying the harm) and treatment (enforcement of remedies)[32] According to Schrepel, computational techniques can enable better, faster analysis of the deluge of market data and help competition agencies detect and understand patterns of anti-competitive behavior.[33] Furthermore, agencies are under both time and resource asymmetries when it comes to review voluminous data involved in merger control. Computational tools can help with both information management and analysis of merger proposal data.[34] Finally, computational antitrust can also help in improving the design, monitoring and evaluation of antitrust policies. First, through better retrospective analyses of policy (as well as enforcement and merger control decisions). Second, through the use of computation to simulate the effects of proposed policies and predict their effects on the cost-benefit equation.[35]

This development can be considered the next stage of development of competition law enforcement. With Antitrust 1.0 involving purely legal reasoning based on statutes; Antitrust 2.0 - law along with econometrics and industrial organization; Antitrust 3.0 - the addition of computational techniques to enhance enforcement workflows.[36]

Computational anti-trust offers a way to cope with the growing complexity of enforcement. However the use of probabilistic AI into a legal system built on the presumption of innocence, due process (e.g. duty to give reasons) gives rise to a fundamental tension. The use of computational techniques is necessitated by the need to enforce competition norms against algorithms (i.e. algorithms need to be subjects of enforcement actions) Especially in the context of algorithmic collusion - where competitors use the same or similar pricing algorithms, which lead to higher prices without any human meeting or agreement.

[32] Thibault Schrepel. "Computational Antitrust: An Introduction and Research Agenda". In: *Stanford Journal of Computational Antitrust* 1 (2021), pp. 1–15. DOI: 10.51868/1. URL: https://law.stanford.edu/wp-content/uploads/2021/02/Computational-Antitrust-%E2%80%93-1.pdf (visited on 11/24/2025).

[33] Schrepel and Groza, *Computational Antitrust Worldwide: Fourth Cross-Agency Report*, at 5-6.

[34] Ibid., at 7-8.

[35] Ibid., at 9.

[36] Thibault Schrepel, "Computational Antitrust", at 2.

The use of computational law techniques coincides with the "algorithmic turn" of market transactions (use of pricing algorithms, automated agents). The approach can be oriented towards monitoring and regulation of these algorithms. For example, when a human programs algorithms to execute a collusive strategy (e.g. match the highest price), the liability here is straightforward under Art. 101 TFEU, as the algorithm is merely a tool for implementing a human agreement.[37] The law can be uncertain, however, for autonomous algorithms that monitor the market and react to competitor's prices. If these algorithms through a "learning" function, discover that keeping prices high is the optimal strategy for profit maximization they may coordinate tacitly. Algorithmic collusion becomes more common, traditional evidence may not exist, creating a legal vacuum where a violation is purely computational (observable in the data) but the legal standards provide human-intelligible reasons that the "black box" cannot provide. Under this scenario, algorithms can effectively achieve the outcome of a cartel without the legal elements of one.[38] Computational law can aid in enforcement and monitoring of "compliance by design", where firms program their algorithms to actively avoid anti-competitive outcomes.[39]

Finally, Schrepel outlines the challenges of Computational Antitrust (parts of which are confronted by the present study): First, the coding of antitrust laws and rulings in order to create efficient, consistent methods for assessing compliance with them. Such encoding can be the basis for developing tools that can assist agencies n automating enforcement and merger control procedure.[40] The second challenge to computational antitrust - is to determine the nature of the data to be collected for these tools, for what types of analysis. Logical analysis (the approach taken by the encoding of this work) for example, is a likely approach for retrospective analysis and the making of

[37] Maria Giacalone. "Algorithmic Collusion: Corporate Accountability and the Application of Art. 101 TFEU". in: *European Papers - A Journal on Law and Integration* 9.3 (2024), pp. 1048–1061. ISSN: 2499-8249. DOI: 10.15166/2499-8249/798. URL: https://www.europeanpapers.eu/en/europeanforum/algorithmic-collusion-corporate-accountability-application-art-101-tfeu (visited on 11/28/2025), at 1052.

[38] Art. 101 of the TFEU requires "agreement" or "concerted practice" which implies a meeting of minds. If algorithms independently learn to coordinate prices without any human instruction to collude (collusion by code) - this may fall outside EU law. ibid., at 1053.

[39] Ibid., at 1060. This assumes that the exact technical parameters of what constitutes a pro-competitive algorithms is undefined.

[40] Thibault Schrepel, "Computational Antitrust", at 12.

counterfactuals.[41] Finally, the third challenge relevant for this work is determining the appropriate role for these computational tools in the decision making process of agencies.[42]

2.2.4 Computable Contracts

Wolfram on the other hand sees computational law as part of a larger trend towards abstraction and formalization, not just in the law but in all spheres of human activity: The development of language and systems of writing themselves can be thought of as an initial step in this trend.[43] Written language enabled law to have coherent, codified forms, as well as a record for deciding ground facts and establishing precedent. While fields such as the natural sciences have progressed in terms of abstraction and formalization to build more intricate systems of knowledge, the law has lagged behind. What is required is the development of a symbolic discourse language for communication of legal and normative concepts, not just with each other, but with computers. Wolfram uses contracts as the starting point for demonstrating the feasibility of formalization and its consequences: A contract in computational form can be defined relative to a set of underlying laws, that serve as the built-in functions of his hypothetical “symbolic discourse language".[44] Once a contracts are converted into a program written in a symbolic discourse language, we can perform all sorts of operations - like determining if a contract implies a certain outcome (or is contrary, complimentary with other contractual and normative commitments).[45]

[41] Ibid., at 12-13.

[42] Ibid., at 13-14.

[43] Stephen Wolfram. “Computational Law, Symbolic Discourse and the AI Constitution”. In: *Data-Driven Law: Data Analytics and the New Legal Services*. Ed. by Ed Walters. Red. by Jay Liebowitz. Data Analytics Applications. Boca Raton, FL: CRC Press Taylor & Francis Group, 2019, pp. 144–174. ISBN: 13: 978-1-4987-6665-4. URL: https://writings.stephenwolfram.com/2016/10/computational-law-symbolic-discourse-and-the-ai-constitution/ (visited on 01/14/2024), at 156.

[44] Ibid., at 157.

[45] Wolfram also notes that even if built on a computational strata, our computable contract may still come across a problem of formal undecidability. i.e. there is no guarantee, even with a formal problem definition, that it is susceptible to solution based on systematic finite computation. ibid., at 158.

One consequence of the computability of contracts is that these can then take in inputs from a variety of sources (including other computable contracts), in order to resolve automatically.[46] The usefulness of computational contracts will depend on what the inputs are (and their quality, availability).[47] Some of those inputs will be natively computational - like the latency of a system, or the amount of digital currency present in an account. as more and more transactions become online, these type of inputs will be more useful. However, not every input is born digital, and will need to take into account the state of things and events in the outside world. Digital analogues may be available for some of these inputs, such as GPS coordinates for location, as well as IoT sensors for basic physical measurements(weight, temperature, vibration). For more complicated inputs that are required to produce legal consequences (i.e., is a person dead, or did the delivered goods meet the stipulated quality standards) may require either manual input, or the use of AI.[48]

Computational contracts can be self-enforcing, automatically running just like any software process. A counterweight to this autonomy is the trustworthiness of the computed determinations - i.e. how can we be sure that the computation was reached with integrity (i.e. that the process was neither hacked nor erroneous)?[49] Wolfram imagines that existing contracts written in natural languages can be translated into a symbolic discourse language, which should be complete and expressive enough to describe ethical and normative systems.[50] More likely, however, new contracts can be written directly into the symbolic discourse language.

[46] Similar to, but at a higher scale and level of complexity, the automated "working out" of options transactions in electronic markets. Wolfram, "Computational Law, Symbolic Discourse and the AI Constitution".

[47] These inputs could include: 1. Intrinsic - Such as the computer's date and time; 2. Extrinsic - Publicly accessible data like stock price, temperature, or a seismic event (which can be consolidated or mediated through something called an "oracle" that a computational contract has access to); 3. Non-public information - Humans, or machine learning systems can intervene ibid., at 162-163.

[48] The AI component, which may use machine learning techniques, will be less transparent and subject to algorithmic biases - just like human determinations which can also opaque and biased. The AI determinations, on the other hand, are at least more amenable to systematic analysis. To ensure its reliability, this component can be subjected to a security-risk model of evaluation and subject to cycles of exploit and patching ibid., at 160.

[49] Technologies such as blockchain, encryption, as well as regular audits may help address these concerns. ibid., at 162.

[50] In cases of ambiguity, the translator-programmer can select an authoritative version, or provide alternative interpretations. ibid., at 163-164.

Adding a computable element to contracts is a way to deal with the growing cost and complexity of transactions. Wolfram also suggests that binding agreements, expressed in computational terms, may also be the means through which we can communicate normative constraints to Artificial Intelligence.[51]

2.2.5 Approaches to Implementing Computational Law

There are two distinct approaches to implementing computational law systems:[52] 1. The **semantic or symbolic approach**, which focuses on structures and the meaning of legal concepts; 2. The **syntactic or connectionist approach**, which depends on algorithms to tease out emergent connections from the data.Taking one approach over the other can involve tradeoffs in terms of accuracy, reliability, and cost.

Semantic approach The semantic approach aims to represent the normative content of laws, rules, contracts into machine-readable formats. At the practical level, it involves humans looking at the raw legal text, pre-processing the text by looking for the meaningful elements, and encoding them into a structured format (e.g. markup languages based on the eXtensible Markup Language, or XML). The primary use-case for the approach is the translation of legal text into something "computable" (i.e. amenable to processing by computers) in order to facilitate "straightforward interpretation".[53] This corresponds to the early work from (Ashley, 2001), which suggests that for computers to undertake analytical legal reasoning, we would require mechanisms for both knowledge representation (to capture relevant aspects of legal knowledge) as well as inferences (algorithms that would allow a program to use the knowledge representations to solve problems).[54]

[51] The constraints we need to enforce on AI will have to be natively computational, since the behavior and possibilities of AI may be too broad, too complex to be expressed in natural language law. It would also not be enough to make it ingest the whole corpus of the law in natural language text as training data: It may be dangerous to give A.I. vaguely couched natural language constraints, since by default it will only literally follow the letter of the law, and exploit ambiguity to achieve hard-coded goals. ibid., at 167.

[52] Andersson, "Computational Law: Law That Works Like Software", at 10.

[53] By straightforward interpretation Anderson means a method that excludes heuristics and other imprecise shortcuts ibid., at 11.

[54] For Ashley, knowledge representation comes in the form of a conceptual hierarchy of legal information: 1. The first level composed of cases and their facts; 2. The second level of "factors", or stereotypical fact patterns; 3. The third

The semantic approach shows greatest promise in contexts that have little ambiguity.[55] An example would of such setting would be the law governing financial options - particularly the determination of the expiration date. Translating legal text into a data structure would involve parties defining a standard as to what constitutes valid, meaningful entities and relationships. In the case of the previous example regarding financial options, parties would have to agree on what constitutes an option, and what an expired date means in relation to the former. Ambiguity or divergent understanding of terms may be resolved through:[56]

1. **Inference from unstructured information** - The designers of the system can use unstructured information such as the web or other collections of text, and apply heuristics to derive the appropriate meaning. Andersson notes however that computers cannot, on their own, start with unstructured information to arrive at authoritative answers, especially to legal questions that do not have predetermined answers;

2. **Deductions from structured knowledge** - Designers of a legal reasoning system would be required to determine in advance where the possible ambiguities lie and how these ambiguities. The problem with this approach however is that in representing the knowledge, one has to make design choices that can limit fidelity to the real world represented. Furthermore, as this encoded structure becomes more extensive and complicated - the issue can become how it can be analyzed efficiently.[57]

Within these general approaches to resolving ambiguity are intermediate approaches: 1. The deterministic approach - where the computational system arrives at a single finite state. Users can or may be required to intervene during intermediate stages. While this may simplify computational legal reasoning, it detracts from the ultimate goal of making law behave more like software. 2. On

level with elements of legal claims; 4. The fourth level, with the legal rules and principles. Kevin Ashley et al. "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers". In: *University of Chicago Law School Roundtable* 8.1 (2001), pp. 1–28, at 2.

[55] Andersson, "Computational Law: Law That Works Like Software", at 11-12.

[56] Ibid., at 15-17.

[57] Ibid., at 17.

the other hand, a non-deterministic approach is purer and more practical. This involves comparison of the facts (with their attendant ambiguity) with precedent, an automated identification of strong and weak points of argumentation, and then presenting all alternatives in the final output. The user will only intervene at the final stage by adjudicating between alternatives presented.[58]

To carry out legal reasoning under this approach would require tools for 1. knowledge representation (to capture relevant aspects of legal knowledge); and 2. inference mechanisms - the algorithms that allow a program to use the above knowledge representations to solve problems.[59]

Recently, open standards based on the eXtensible Markup Language, such as Legal XML, have been adopted by the legal technology community to facilitate the encoding of legal texts into machine-readable formats.[60] The encoding and analysis carried out by this work will itself use technologies based on these formats.

Syntactic approach In the syntactic approach,[61] legal concepts are transformed into data structures which are then subjected to symbolic manipulation. The emphasis of this approach is in trying to mimic the process of reasoning, rather than the content of the law.[62] The approach is more heuristic, and less reliant on the formal structures of the law. The syntactic approach is more flexible, and can be used in a wider range of contexts. However, it is also more prone to errors, and may require more resources to implement. The syntactic approach can be most appropriate in areas of the law that involve complex analysis of facts, subject to interlapping rules.[63]

[58] Ibid., at 17.

[59] Algorithms at the inference level will use the information at the knowledge representation level. E.g. The inference mechanism takes as input a problematic situation, compare it to other cases. Based on this analysis, can draw inferences on how the problem should be decided. Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 2.

[60] Andersson, "Computational Law: Law That Works Like Software", at 20.

[61] Also referred to as the connectionist approach

[62] Andersson, "Computational Law: Law That Works Like Software", at 17-18.

[63] An example would be cause of action analysis for copyright infringement, which would involve separate determinations for: 1. Copyright eligibility; 2. Exceptions and affirmative defenses. ibid., at 18.

Legal texts need to be translated into formal representations. Engineers will need to ensure that formal representations accurately reflect informal legal texts, even as the legal regime is constantly evolving. If not, we have a state of incongruity.[64] We can try to resolve incongruities through the following approaches:[65]

1. **Harmonization** - Refers to any strategy for ensuring congruity after finalization of the legal text. This can involve post-facto interpretation or amendment to ensure that the electronic representation fits the legal text. This approach may be considered a temporary bridge for the current mass of legal text, to be abandoned in the long term in favor of unification;

2. **Unification** - Involves rewriting laws in a way that is syntactically coherent for computers. Thay is, bound by the same formal rules as a programming language.

The algorithmic approach can be implemented through almost any modern programming language.This goes with the assumption, however, that the legal question being confronted is computable - i.e. that it is possible to create a program that can answer a given legal question based on inputs to the system.[66]

Based on the limitations of the above approaches, Andersson asserts that *post-hoc* disambiguation is intractable. Instead, he proposes the use of specialized tools to prevent ambiguity *ex-ante*, by 1. imposing constraints on how users construct legal documents; and 2. translating these constrained choices into machine readable format.[67]

A language for representing rules computationally must satisfy the following requirements: 1. It should accomodate semantically-rich rulesets even as it formally models behavior; 2. It should be grounded in logic, since we are concerned with performing automated analysis of the rules; 3.

[64] Andersson, "Computational Law: Law That Works Like Software", at 18-19.

[65] Ibid., at 19.

[66] A further discussion on the definition of computability, and the computability of law, can be found in Chapter 3.

[67] For example, the authoring tool for financial contact will require an expiration date and price, with input validation constraints (e.g. expiration date must be a date in the future, and price must be a positive number with a currency.) Andersson, "Computational Law: Law That Works Like Software", at 16-17.

Finally, it should be capable of representing sequences of actions, and behavioral constraints on those action.

2.2.6 Relationship with Current AI Implementations

Computational Law appears to overlap with artificial intelligence in terms of function. Explicitly encoding the rules of law and legal reasoning can be considered an application of **declarative artificial intelligence**,[68] an approach to AI that focuses on the representation of knowledge and reasoning. Recent developments show great promise from the connectionist approach to AI, which focuses on the use of neural networks and deep learning. The latter approach has been used to develop large language models (LLMs) such as GPT, which can perform a variety of tasks, including generate text that reads like plausible legal reasoning. Given this state of affairs, will it not be better to develop LLMs to perform the tasks of legal analysis and evaluation? One might suggest simply feeding ChatGPT the corpus of existing law, and expect it to perform legal reasoning. In some ways, the declarative approach is similar to the training of a machine learning model to perform the same function. The difference is that the former is a more explicit, transparent, and controllable process. The latter is more opaque, and the results are less predictable. Thus, for mission-critical domains like law, there is merit in explicitly encoding rules over LLMs and deep learning. A fuller discussion of practical and ethical limitations of these systems will be covered in Chapter 9. As a useful contrast to the approach that will be taken by this work, the more critical limitations are discussed below:

No ground truth In any conversation or task, humans rely on an internal model of the world - a theory of mind[69] from which it can draw meaning (including the motives of other humans).

[68]The division between approaches to the computational law described above maps perfectly to the division in artificial intelligence: The semantic approach corresponding to Rules-Based AI - which encompasses Semantic Web, Ontologies, and Knowledge Graphs, rooted in symbolic logic and promises precision, explainability and structure. On the other hand, the syntactic approach corresponds to that taken by the current batch of Generative AI - including those systems based on machine learning and neural networks. The approach has flexibility, creativity, and the ability to process unstructured data at scale.

[69]Theory of mind refers to the uniquely robust human cognitive capacity to attribute mental states (e.g. beliefs, intentions, emotions, and knowledge) to oneself and others. David Estes and Karen Bartsch. "Theory of Mind:

Humans have both the capacity and the institutions to derive meaning. On the other hand, the current consensus is that while generative AI may have "emergent" capacity for reasoning, AI does not have a true theory of mind,[70] relying only on surface-level statistical regularity of text (which is itself replete with psychological descriptions)[71] - In an ordinary conversation or task, humans rely on an internal theory of the world, a theory of mind for the entities that it is dealing with. We have a phenomenology. We are still not sure if this can be done for AI's. Certainly this is not what happens with LLMs. These models do not have ground truth. Without the capacity for ground truth, LLM's can spiral into delusions - making their applications unsafe for mission-critical applications. This limitation appears to be insurmountable since AI lacks embodied grounding - it is not capable of physical interaction with the world at large, in order to observe and confirm the truth for itself, and this correlate that data with intent and meaning. The consequence of lack of grounding is that current AI implementations only manipulate tokens without mechanisms to connect those symbols with physical reality through perception and sensory-motor experience.[72]

Prohibitive costs As will be discussed at greater length in Chapter 9, the current implementations of generative AI are expensive to build, train, and maintain. LLM's are expensive - These systems are expensive to build, train, and maintain, even on a per-query basis once everything is set up. There is a large fixed cost associated with research and development, building the data centers, outfitting them with processors (usually customized Tensor Processing Units or Graphic Processing

A Foundational Component of Human General Intelligence". In: *Behavioral and Brain Sciences* 40 (Jan. 2017), e201. ISSN: 0140-525X, 1469-1825. DOI: 10.1017/S0140525X16001618. URL: https://www.cambridge.org/core/journals/behavioral-and-brain-sciences/article/theory-of-mind-a-foundational-component-of-human-general-intelligence/D36195EE2D712887FE6DFDC83E6BA16B (visited on 05/28/2026); See also: Bertram F Malle. "Folk Theory of Mind: Conceptual Foundations of Human Social Cognition". In: *The New Unconscious*. Ed. by R. Hassin, J.S. Uleman, and J.A. Bargh. New York: Oxford University Press, 2003, A theory of mind is a prerequisite of complex behavior we attribute to intelligence: language acquisition, strategic social interaction, objective moral reasoning, and cultural pedagogy.

[70] Michal Kosinski. "Evaluating Large Language Models in Theory of Mind Tasks". In: *Proceedings of the National Academy of Sciences* 121.45 (Nov. 5, 2024), e2405460121. ISSN: 0027-8424, 1091-6490. DOI: 10.1073/pnas.2405460121. URL: https://pnas.org/doi/10.1073/pnas.2405460121 (visited on 05/28/2026).

[71] Ruirui Chen et al. "Theory of Mind in Large Language Models: Assessment and Enhancement". In: *Proceedings of the 63rd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*. Vienna, Austria: Association for Computational Linguistics, July 2025, pp. 31539–31558.

[72] Goonmeet Bajaj et al. "Grounding From an AI and Cognitive Science Lens". In: *IEEE Intelligent Systems* 39.2 (Mar. 2024), pp. 66–71. ISSN: 1941-1294. DOI: 10.1109/MIS.2024.3366669. URL: https://ieeexplore.ieee.org/document/10510670/ (visited on 05/28/2026).

Units), training their models based on large corpus of text.[73] Operating these data centers will require huge amounts of power. This will lead to a large carbon footprint, as well as significant water consumption for cooling requirements.These are also expensive to retrain. If a Large Language Model gets "poisoned" by malicious input - what are the ways to mitigate it? How can one "nudge back" if the mechanism is hard to trace and hard to explain. Fixing it will require a lot of resources and the solution might not be durable (It could also affect the accuracy and the responsiveness of the model).

Despite the initial wariness about the costs and consequences of large language models, their growing sophistication is compelling. Recent literature suggests that knowledge graphs can be embedded into large language models, making the latter more efficient, more attuned to “ground truth", and therefore more reliable. Since both argumentation frameworks and proposition networks can be framed as extensions of the information contained in knowledge graphs, it may be possible to combine these approaches as well.

The connectionist approach to AI and declarative computational law approaches, although distinct, can still converge and reinforce each other: Data-driven LLMs can inform and enrich our techniques for encoding laws. The more varied the laws we have to encode, the more diverse the rules that can form the basis of a computational law system, and the more accurate and relevant will its determinations be. At the same time techniques of logic used by computational law can enhance algorithms used by the data-driven approach by providing a more nuanced view of legal knowledge and legal reasoning.

It may be possible to combine LLM’s with the declarative approach in mutually beneficial ways: More on the feasibility and applications of a hybrid approach (called the Neuro-Symbolic AI), is provided in Chapter 9 below. To summarize, the convergence can works by leveraging their

[73] United Nations International Computing Center. *A Frugal AI Framework to Measure Your AI Portfolio as a Strategic Asset*. United Nations, Dec. 2025. URL: https://www.unicc.org/wp-content/uploads/2025/12/From-Total-Cost-of-Ownership-to-Social-Impact-by-Frugal-AI-Hub-and-UNICC-AI-Hub.pdf.

respective strengths and complementing each other's gaps: While generative AI excels at processing natural language fluency and navigating the semantic nuances of legal text, it is inherently suffers from non-deterministic outputs, hallucinations, and a lack of traceable reasoning. On the other hand, the symbolic,rule-based AI approach to which this work contributes can provide deterministic outputs as well as traceability. Such systems, however, require manual encoding rules, which can result in a bottleneck for knowledge acquisition and inference generation.

Wolfram suggests that a sufficiently trained machine model can interact with norms defined in a symbolic discourse language: Overall goals and standards can be defined in the symbolic discourse language, while the machine learning model can fill in implementation details.[74] Machine learning models can also be trained to convert a huge corpus of legal texts into a initial encoding in the symbolic discourse language.[75] The connectionist model can act as a semantic parser, ingesting the raw unstructured text of the law, and formalizing it into machine readable symbolic code, which can then be processed by a reasoning engine.

2.3 Computational Law Examples

Let us imagine a business with the following configuration of employees and offices:[76]

John manages Ken	John is in office 22	John is male
John manages Kat	Kat is in office 24	Jill is female
Jill manages Mark	Ken is in office 22	Ken is male
Jill manages Mike	Kat is in office 24	Kat is female
		Mary is female
		Mike is male

Logic and programming constructs allow us to: First - use of variable to represent an arbitrary number of entities (X, Y, Z for employees and offices). Second - use of logical operators to express

[74] Wolfram, "Computational Law, Symbolic Discourse and the AI Constitution", at 165-166.

[75] Wolfram believes that machine learning models are likely to have limits in how they model concepts (such as the notion of space). Thus, human intervention will always be required in encoding laws and norms ibid., at 157.

[76] The following examples are from Genesereth, *Computational Law: The Cop in the Back Seat*, at pp. 3-5.

relationships between any of the above (not, and, or, if-then). These "representational extensions" allow us to define new relations in terms of existing relations:

> If X is in office Z and Y is in office Z and X and Y are *distinct*, then X is an *officemate* of Y.

In addition to merely describing entities and their relationships, we can encode rules and regulations through the use of these programmatic tools. We can ascribe the attribute of illegality to some facts or relationships:

> If X manages Y and X is an officemate of Y, then that is *illegal*.

Within a given set of facts (entities and relationships) and rules (deontic assertions) it may be possible to derive other conclusions:[77] These patterns of reasoning are called "inference rules" or rules of inference. Iterative use of inferential reasoning can generate all logical conclusion (facts and rules) from within a given set of premises (facts and rules).

Example of inferential rules discovery and compliance check:

- John is in office 22
- Ken is in office 22
- John is an officemate of Ken
- John manages Ken and John is an officemate of Ken
- John is not Ken
- That is illegal

[77] According to Genesereth, 2015: "...by matching facts and conclusions of rules to the conditions of other rules and asserting their conclusions"

We can invert this reasoning working backwards and arranging facts to avoid illegalities

- John is in office 22
- Jill is in office 24
- Ken is in office 22
- Kat is in office 24

The inference rule discovery process can be extended to look for inconsistencies within a set of regulations. For example: We might require every project to have managers and subordinates, and no manage have a subordinate who is also an officemate. This might be inconsistent with a subsequent rule requiring special projects personnel be housed in a common work room. Compliance checking (through automated legal reasoning) can feed legal planning and regulatory analysis.

2.4 Advantages of the Computational Law Approach

Computational law can cover a range of possible uses cases, often derived on the promise of being able to automate legal determinations at a higher quality and less cost - without the human biases and limitations of human functionaries.[78] Based on the above definitions of computational law, and depending on the horizon of technological development considered, possible computational law implementations fall into two major categories:[79]

1. **Specific computational law** - Such as simply confirming the presence of necessary elements of a cause of action, as in a checklist.

2. **General computational law** - Capable of making nuanced determinations if presented with a complex fact pattern within a specific (or even several) legal regimes.

[78] Simon F. Deakin and Christopher Markou. "From Rule of Law to Legal Singularity". In: *Is Law Computable? Critical Perspectives on Law and Artificial Intelligence*. Oxford: Hart Publishing, 2020, pp. 1–29, at 5.

[79] Andersson, "Computational Law: Law That Works Like Software", at 6.

The encoding and inference to be covered by this work belongs to the first category of specific computational law - checking encoded laws against the requirements of the OECD guidelines.

Among the bolder visions of general computational law imagines a scenario where trends of massive growth of available data, translating to a higher quantification of observable phenomena, and better capabilities of inference, through more accurate analysis, converge into a “legal singularity" where legal uncertainty is totally eliminated.[80] Even without totally buying into this theoretical scenario of perfect legal knowledge, and regardless of the implementation, computational law can enable the following specific applications:

2.4.1 Enabling applications for automated legal reasoning

Rule/argument generation Computational law could lead to the development of applications that are capable of causal inference in law. That is, being able to know what follows from particular legal premises. Granted, that there is more to legal decision-making and legal reasoning than the mechanical application of some fixed set of legal rules,[81] such systems can still help even experts navigate through premises and consequences of complex, interlocking regulations, which can otherwise be tedious, time consuming, prone to errors.

Assuming facts and rules can be well-defined, a computational process can derive other applicable rules.[82]

Legal outcomes prediction Related to the generation of other feasible rules is the prediction of legal outcomes. Given a state of affairs, sufficiently described, a computational law system

[80] Deakin and Markou, “From Rule of Law to Legal Singularity”, at 5.

[81] Kowalski and Sergot, “The Use of Logical Models in Legal Problem Solving”, at 212.

[82] The simplest and oldest attempts at computational law applications often involve the mapping of legal rules into logical rules. This approach can be useful if the problems are stereotypical and clear, i.e. there is little to no context dependency that will make the application of rules contingent. Branting characterizes UCC-related problems as those most likely to be amenable to the approach. While those that involves broad standards, such as “reasonable care" is not Ashley et al., “Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers”, at 14-15.

can determine the legal consequences that are likely to follow from factual and legal premises.[83] Software can be particularly useful for chasing down the consequences arising from a particular human interpretation of the law (such as an administrative interpretation, which holds until corrected by the court or replaced by a new administrative issuance).[84]

A computer can treat factual circumstances in the present as data, while the applicable rules can be represented as algorithms that can process such data to determine likely results. Predictive systems may be adopted by legal practitioners, since advising their clients may often involve predicting the outcome of legal controversies. Having computational systems that look at the data from an uninterested perspective may be helpful since lawyers' calculations may be skewed by their optimism, or by an overestimation of their own skills. This can result in suboptimal outcomes for their clients, the courts, and society as a whole.[85] Computational law systems can not only show the consequences that will flow from premises, but it can also explain (i.e., present proofs), identifying the interpretation of the law incorporated in its reasoning. The human lawyer is expected to check if the interpretation is appropriate, and base further action on the prediction.[86]

Document processing Finally, computational law systems can serve the requirement for the drafting, preparation, and filing of legal documents. The ability to infer rules and predict outcomes can be combined with exiting sophisticated models (such as those provided by natural language processing) in order to create drafts.[87] Policymakers can run the computational representation of proposed legislation and run them through a suite of tests such as: 1. Consistency with goal; Effect in alternative factual scenarios; 3. Consistency with other norms.[88]

[83] Kowalski and Sergot, "The Use of Logical Models in Legal Problem Solving", at 203-204.

[84] Kowalski likens this testing the legal consequences based on a particular interpretation to an engineer measuring bridge tension in a particular component. ibid., at 213.

[85] Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 15-16.

[86] Kowalski and Sergot, "The Use of Logical Models in Legal Problem Solving", at 213.

[87] Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 16.

[88] This is the general direction and the ultimate goal of the current project. See Kowalski and Sergot, "The Use of Logical Models in Legal Problem Solving", at 210.

All of the above applications can be enabled by a body of formalized legal knowledge and the algorithms to process such knowledge. A computational model of the law can be serve the use cases of different stakeholders in different settings. Furthermore, there is a history (and variety) of computational models trying to respond to the applications described above, with a wide range of capability and corresponding costs.[89]

2.4.2 Enabling legal self-help

The availability of rule detection and automated legal analysis can enable legal self-help - actors structuring/planning their activities (especially electronic transactions) to be legally valid/compliant.[90] Logical representations can make it possible to derive common baseline rules, or discover bridging rules (or the exact points of divergence). This will then make it easier to have analyze cross-border contracts, or do comparative legal analysis.[91]

Love and Genesereth maintains that such systems and self-help only extends to reducing transaction costs for legal compliance and does not mean that parties can appear *pro-se* in instances of conflict. The forum of computational law is within enterprises, and not courts.[92] Enterprises and agencies, due to their functions often operate in an environment where legal text is transformed into explicit operational requirements - through forms, checklist or hard coded expert criteria.[93] For example, in

[89] Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 16.

[90] Genesereth and Love, "Computational Law", at 206.

[91] But see Alarie, "The Path of the Law", at 1-3. The following discussion on capabilities and applications of Computational Law systems may be considered modest, especially when compared to Alarie's vision of a "legal singularity" brought about by greater computational capabilities and availability of data. Using tax law as an example, some of the transformations brought about by this convergence include: 1. Improved dispute resolution and access to justice - A shift fr om standards (broad, adjudicated ex post facto) to a more complex but query-able system of rules (that are knowable ex-ante), 2. More complete specification of tax law. The emergence of a more complex regime that is nevertheless capable of precision, coherence, and distribution of burden (at least compared to the current system) Legal uncertainty can be eliminated under such a regime, and legal disputes will be rare. Agreed upon or discovered facts can be readily mapped to clear legal consequences.

[92] See. Citing Loftus and Wagenar: "Optimism is rewarded...The most successful trial lawyers are those whose estimates are least realistic, that us, are most overly optimistic...This means that as an institution, courts are rewarding behavior that isn't optimally beneficial to the system as a whole... Genesereth and Love, "Computational Law", at 206.

[93] Bao Chau and Michael Livermore. "Computational Legal Studies Comes of Age". In: *European Journal of Empirical Legal Studies* 1.1 (May 13, 2024), pp. 89–104. ISSN: 2004-8556. DOI: 10.62355/ejels.19684. URL: https://publicera.kb.se/ejels/article/view/19684 (visited on 05/29/2026).

the calculation of taxes or determination of eligibility for social benefits, regulatory compliance monitoring. This will be enabled by legal provisions being encoded into computable code. Other fields appropriate for computational law include: Supply chain logistics, decentralized finance, conditional property transfers where legal obligations, rights can be tied to objective, verifiable event triggers.[94]

Genesereth sees potential in embedding computational law applications into software that supports workflows that are subject to legal, regulatory requirements - e-commerce, data privacy, etc. Genesereth points to Project Calc (A Stanford CodeX project under Harry Surden), which integrates into CAD software used by architects, routines for checking compliance with rules such as: building codes, environmental rules, accessibility laws.[95]

We can embed computational law applications in devices such as cellular phones, car dashboards, smart glasses so that they can provide legal guidance at the point of decision. For example: An app that not only identifies the flower the picture of which you took, but also informs you that you should not pick it up. Compared to simply publishing an overwhelming mass of laws (often in a language inscrutable to the public) digitally-mediated legal determinations can help make the notice requirement of due process more meaningful.[96] For automobiles (whether manned and unmanned), in addition to basic functions such as navigation and collision avoidance, the system can help compliance with legal requirements such as: a. speed limits; b. whether or not a street is one way c. whether u-turns are allowed; d. what areas allow parking.[97]

Different systems may be required for different participants in the legal system, with sophistication and capability scaling to the requirements of users along this continuum. Ordinary users may

[94] "Many legal processes can be viewed as motion from one state to another. For example, one can move from the initial state of 'no contractual obligation' to the intermediate state of 'valid offer' to the ultimate state of 'contract'" Jeffery Atik and Valentin Jeutner. "Quantum Computing and Computational Law". In: *Law, Innovation and Technology* 13.2 (July 3, 2021), pp. 302–324. ISSN: 1757-9961, 1757-997X. DOI: 10.1080/17579961.2021.1977216. URL: https://www.tandfonline.com/doi/full/10.1080/17579961.2021.1977216 (visited on 05/29/2026), at 312.

[95] Genesereth, *Computational Law: The Cop in the Back Seat*, at 6-7.

[96] Ibid., at 7.

[97] Ibid., at 7.

only need answers for simple scenarios. Lawyers may require argument generation based on legal premises and factual scenarios. Judges can use similar systems, but for evaluating the basic validity of arguments and precedents.[98] Finally, legislators and policy makers can use computational tools in order to evaluate proposed rules against other norms, as well as predict the impact of draft laws.[99]

2.4.3 Normative reasoning of autonomous agents

As more transactions in important aspects of our economic and social lives are handled by software agents, it becomes necessary to ensure that such agents are not just bound by their functional requirements, but are well-behaved, by being governed by norms. Boella envisions multi-agent systems, where agents (both human and software based) can interact with each other, and such interactions are governed by norms. Such norms allow for the possibility that actual behavior may at times deviate from the ideal state set by the norm (e.g. violation of obligations, agents' rights).[100]

2.4.4 Consistency and predictability

The utility that can be derived from the computational approach is compelling enough to warrant its pursuit. The more obvious advantages come from the speed and reliability of computers, as well as their ability to retrieve relevant legal text from memory.[101] However, some of the more fundamental advantages to the profession can be indirect: The rigorous structured approach of these

[98] Kowalski imagines an example where the encoding of social security laws can be turned into an application for government officials overseeing applications, taking into account the office's internal logic for evaluating claims. At the same time, the encoding can be used for a general version of the application available to the public, using legal knowledge to advise them with their applications. Kowalski and Sergot, "The Use of Logical Models in Legal Problem Solving", at 208-209.

[99] Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 14. The requirements for legal professionals can be further broken down to the following functions: 1. Problem formulation - Formulate the problem in terms of the relevant legal concepts, 2. Retrieval - Gather authorities relevant to the problem as formulated, 3. Problem analysis - Determining the legal consequences that follow from application of authorities to the facts. 4. Prediction - For each of the possible outcomes borne by the analysis - what are the probabilities of each outcome?

[100] A normative system can allow the well-ordered interaction of multiple agents in a heterogenous environment, where there is no centralized architecture to constrain (electronic commerce, virtual communities are possible settings) Guido Boella and Leendert van der Torre. "Regulative and Constitutive Norms in Normative Multiagent Systems". In: *Principles of Knowledge Representation and Reasoning: Proceedings of the Ninth International Conference*. Knowledge Representation 2004. 2004, pp. 255–265, at 255.

[101] Grossman and Solomon, "Computers and Legal Reasoning", at 66.

systems may "mold the thought processes of the lawyer" (and law students) into a more logical pattern, and the extended use and design of such systems will force legal scholars to confront and resolve the ambiguities of the law.[102] Investigations into computerized encoding and analysis can be useful not just for the development of practical application but also for clarifying and improving the process of legal reasoning.[103] Genesereth argues that simply publishing the overwhelming mass of laws, in a form inscrutable to the public is not adequate notice. Computational law, by providing digitally mediated legal determinations can help address this gap.[104]

Representing and analyzing laws with the computational approach can provide certain advantages. It can remove or minimize the degree of legal uncertainty (characterized by radicalization of legal realism, or postmodernism), and make law more transparent and consistent. The casting of law within a formalism can enable advanced analysis that goes beyond subjective inferences of human lawyers. Advanced analytical tools such as simulations, derivations, combinatorics can be applied to bodies of law. Finally, lawyers and legal scholars can have a stable point for discussion, without the ambiguity of language across jurisdictions. This can be a basis for interdisciplinary work, as well as a basis for testability and confirmation.

Wolfram notes that at the immediate level, the conversion of legal constructs into the computational form can give them new capabilities, such as automated annotation of implications, simulation of results, statistics and probability analysis.[105] On the other hand, lawyers and law students can think about these legal constructs at a higher level.[106] It gives rise to to clearer thinking about the law - without the semantic ambiguity, cultural baggage of natural language. Wolfram paints the broader implications of the technology by historical analogy: With growing literacy and the development of technology around the written word - there is a growing trend towards complexity of transactions and their corresponding legal instruments. Having a computational component will lead to even

[102] Grossman and Solomon, "Computers and Legal Reasoning", at 66.
[103] See Kowalski and Sergot, "The Use of Logical Models in Legal Problem Solving", at 217.
[104] Genesereth, *Computational Law: The Cop in the Back Seat*, at 8.
[105] Wolfram, "Computational Law, Symbolic Discourse and the AI Constitution".
[106] Wolfram cites the Sapir-Whorf hypothesis - that is, language can affect patterns of thinking ibid., at 164.

greater levels of complexity.[107]

2.4.5 Economic considerations

Beyond the technical feasibility of these systems, and the intellectual curiosity they may inspire - is there sufficient motivation and necessity for the development of computational law systems? Branting predicts that these systems are needed due to a vast, unmet demand for legal services, particularly in the growing government sector - which will need to navigate an ever more complex legal and regulatory regime in order to make the routine legal determinations necessary to carry out its functions.[108]

Building these systems do not mean starting from scratch, since we can leverage existing data on systems that embody business rules, such as those used in banking or human resources.[109] Computational law just extends this tendency by encoding public instead of private rules.

2.5 Limitations of the Approach

As will be discussed later in Chapter 9.1, there are both fundamental limitations to computation, as well as policy reasons not to employ it in law. Thus, not all of legal reasoning may be subject to translation to a computational model. Instead of a outright substitute to legal reasoning by human experts, computational law is proposed as an aid to a subset of tasks such as those mentioned above (e.g. authority retrieval, argument generation, analysis and prediction).[110] The computational approach is often limited by the following:

1. **Open-texture problem** In the real world where lawyers operate, both the rules and assertions of facts may be open to interpretation.

[107] Ibid., at 165.

[108] Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 16-17 Branting also suggests that these systems can be a form of marketing for legal expertise, i.e. software can handle low-end requirements and lead clients to human legal experts for bespoke work.

[109] Genesereth, *Computational Law: The Cop in the Back Seat*, at 7.

[110] Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 14.

2. **Incongruity with actual legal thinking** Legal decisionmaking seems to bypass explicit reasoning around rules and derive from specific cases, often through analogy.

3. **Incompleteness** Formalization can only provide a finite set of rules with which to analyze complex states of the world as well as its normative environments

Open texture One fundamental problem with computational law is how to square formalisms with the open-texture of the law. The concept of "open texture" refers not just to mere vagueness (which can be addressed on a case-by-case basis), but to the persistent, inherent limitation of language, i.e. that linguistic descriptions of the world will always be incomplete.[111] In the field of law, this corresponds to HLA Hart's conception of the law as being composed of a core of determinate meaning, surrounded by a penumbra of unsettled meaning. It likewise presents a bottleneck for computational law, especially the design of systems based on logical inferences: Unless each term is fully defined, a system can eventually "run out of rules" to deal with which to deal with novel situations. The complexity of the law (and the world it operates in) means that the facts and rules that one wants to encode in a categorial manner will be open to intepretation. Genesereth provides the example rule: "No vehicles in the park". This might be obvious to a human in the community, but problematic for someone trying to define the rule. What is a "vehicle"? Is a bicycle a vehicle? How about a skateboard? Roller skates? What about a baby stroller? A horse?[112] Even assuming that logical formalisms can perfectly model legal reasoning - it may still not be able to account for other equally important influences to legal discourse - political, economic, and sociocultural factors.[113] Subsuming all these other factors will not be trivial, as this will only compound what Hampshire describes the inexhaustibility of description[114] - We will never run out of ways to classify

[111] Mateusz Zeifert. "Rethinking Hart: From Open Texture to Prototype Theory—Analytic Philosophy Meets Cognitive Linguistics". In: *International Journal for the Semiotics of Law - Revue internationale de Sémiotique juridique* 35.2 (Apr. 2022), pp. 409–430. ISSN: 0952-8059, 1572-8722. DOI: 10.1007/s11196-020-09722-9. URL: https://link.springer.com/10.1007/s11196-020-09722-9 (visited on 05/27/2026).

[112] Genesereth, *Computational Law: The Cop in the Back Seat*, at 6. For this matter - What is "the park" - i.e. assuming we limit What are its horizontal (and vertical) borders? Can a helicopter hover at ten feet? One hundred feet?

[113] Deakin and Markou, "From Rule of Law to Legal Singularity", at 5-6.

[114] Any reference to identification of a thing amounts to adding to its description. As anything can be connected to everything else based on other dimensions of meaning, there is no end (or even a starting point) to this process, there is no fixed way of classification and enumeration. Stuart Hampshire. *Thought and Action*. Notre Dame, Indiana:

and refer to things.[115]

Genesereth's suggested response to the open-texture problem is to limit computational law applications to cases where such issues can either be 1. externalized - that is, allow human users to input their judgments on open-textured concepts, through data entry or on-the-fly determinations; 2. marginalized - simply do not use the computational approach in areas of law where there are many open-textured concepts.[116]

The programmatic approach (mapping facts and constructing, deriving inferential rules) can express many types of rules. Some rules however are more complicated. Genesereth refers to prior work from Sergot and Kowalski, et al (1986) which explores the formalization of the British Nationality act as a logic program, through conversion of a text into Extended Horn Clauses.[117] However, some legal texts are not readily formalizable with this approach. Such as: 1. When the applicable rule will depend on a person's subjective belief about the facts/ (e.g. if ". . . the Secretary of State is satisfied that. . .) 2. Some rules are dependent on default states that can change under some circumstances, such as contrary evidence (e.g. ". . . unless the contrary is shown. . . ") and 3. Rules that require reference to other parts of the law, or other laws.

Incompleteness Complementary to the problem of open-texture are fundamental limitations to formal, logical approaches. The limits of formal reasoning means that one will not be able to generate enough explicit, categorical rules for resolving the terms of a legal problem.[118] Godel's theories has stopped any pretense that it will be possible to develop fully axiomatized formal systems, even in mathematics. He also demonstrated how such large deductive systems can fail to

University of Notre Dame Press, 1983, at 20.

[115] According to Hampshire, not only can we not say: "I have identified all the things that there are", we can't even say "I have identified all the things that there are in this room". ibid., at 21-22.

[116] Genesereth, *Computational Law: The Cop in the Back Seat*, at 6.

[117] "A person born in the United Kingdom after commencement shall be a British Citizen if at the time of birth his father or mother is (1) a British citizen or (2) settled in the United Kingdom" ibid., at 5, citing Sergot and Kowalski, et al (1986).

[118] Ibid., at 6.

be self-consistent.[119] Godel's theory proves that any formalized system cannot be both perfectly consistent and perfectly complete. Adding new rules to address the gaps will only lead to more complex systems that are mote vulnerable to new inconsistencies and gaps.[120] By analogy Godel's first incompleteness theorem is also applicable to legal structures (even before prior to formalization) - Thus, it is impossible to create a legal system that is complete in the sense that there is a derivable rule for every factual situation.[121] An ambitious version of the computational law project is the translation of legal text and legal reasoning into computable code. However, Kurt Godel's incompleteness theorems proved that within any consistent, formal axiomatic system, there will always be statements that cannot be proven from within the same system, and thus the system cannot prove its own consistency.

The ultimate ramification of incompleteness is that a fully automated algorithmic legal code can never fully capture the infinite variety of human behavior either decaying into logical contradictions or require reference to external reasoning. We can adopt from this limitation by discarding the goal of a perfectly complete system. Instead, its application relies on paraconsistent logic (such as defeasible logic) which can accomodate conflicting rules without falling into logical explosion. Furthermore, these systems acknowledge their limits - these do not seek to replace legal determinations by humans. Instead, these systems are meant to be assistive in nature - handing off the logically undecideable aspects to human judgment.

What may need to be discarded is the notion of strict legal formalism - the notion that the law can be rendered as a complete and self-contained system of rules. Incompleteness has not hobbled mathematics and science, which has maintained their practical utility and certainty despite having

[119] See also discussion on incompleteness as applied to computational representation of law in Chapter 3 Ernest Nagel and James R. Newman. *Gödel's Proof*. Routledge Classics. London New York: Routledge, 2005. 94 pp. ISBN: 978-0-415-35528-5, at 4.

[120] Il'ia L. Chestnov and Ekaterina G. Samokhina. "The Principle of Relativity in the Post-Classical Theory of Law". In: *Journal of Siberian Federal University. Humanities & Social Sciences* 13.1 (2020), pp. 42–49. URL: https://pdfs.semanticscholar.org/c2e9/a00459b45c8c4315946cbb9bdf540c5be4c4.pdf (visited on 05/27/2026).

[121] John M Rogers and Robert E Molzon. "Some Lesson About the Law From Self-Referential Problems in Mathematics". In: *Michigan Law Review* 90.5 (1992), pp. 992–1022.

incomplete foundations.[122]

While these problems might be insurmountable in some legal domains, Love and Genesereth argue that domains where transactions are electronically mediated can make the problems of computational encoding and analysis more manageable. These systems can be considered more amenable to the computational law approach since: Like other legal domains, have entities and transactions that are subject to a system of rules (statutes, regulations, policies). The transactions in these systems are semantically rich - they are well-defined through documentation, code, or system constraints (they also note the industry's move towards semantic data) The information gap problem (when it comes to factual determinations) - is also addressed in these domains, since within these systems, each transaction (and agents involved) can be logged and verified.Finally, these domains are also the most likely users and beneficiaries of computational law systems.[123]

Incongruity with legal reasoning Another possible limitation of the computational approach is that not all legal reasoning is characterized by the formal logical methods employed in programming. As aptly put by Edwina Rissland, et al.: "Law is not a matter of simply applying rules to facts via modus ponens". Many legal determinations are not made from deducting from general principles but inducing from specific cases.We can't map enough deductive rules from a given body of law. Genesereth maintains that since computational law emphasizes deductive reasoning, it cannot be applied to instances of legal determinations that require analogic or inductive reasoning.[124] The use of analogical reasoning is a special problem for computers, since it will require discovering (or event constructing) the relevant principle that establishes that the cases are "similar". While computers can exhaustively search through a given set of predetermined rules that can establish similarity.

Other obstacles to formalization can arise from the ways law is formulated in the first place: 1. Legislation is not always coordinated, since they arise from different contexts (e.g. different

[122] Ibid.
[123] Genesereth and Love, "Computational Law", at 205-206.
[124] Genesereth, *Computational Law: The Cop in the Back Seat*, at 6.

historical settings that confront different problems)2. Legislation has gaps - some entities, actions, relationships, are not covered by any rule 3. Legislation may overlap, or be inconsistent with each other.Genesereth is convinced, however, that since the publication of Sergot, et al., many of the difficulties presented have been overcome by extensions to the language and reasoning of computational law.[125]

Finally, there is some doubt as to whether or not computation can adopt the kind of analogical reasoning often used in legal interpretation. Analogy does not involve merely enumerating similarities from a given set of criteria. Reasoning by analogy does not proceed from premise to conclusion, but is based on the discovery (or even creation) of evaluative principles from which one can assert that one case is similar to another.[126] The search space for such principles may be infinite, given that humans can invent new ways to draw similarities between one category and another. The ability to discover new analogies can also be based on the human experience of being embodied, sensate, and embedded in a culture - attributes that a computer may never have.

Other Countervailing Factors Aside from the more abstract objections to the notion that computational law can be readily adopted - some comment must be given to practical concerns. Modeling the law into computational terms is not an intuitive course of action fro legal institutions whose training and traditions are steeped in natural language. That is, a legal practice that is based on the historical (even cultural) meaning of words, and maintaining a balance between certainty and ambiguity. Furthermore, there is an associated cost not just to the design and deployment of computational law systems, but also ensuring that these systems are integrated not just to practical workflows for lawyers and judges, but also legal education. Institution will require additional expertise, as well as resources to fund the development costs of these systems. As a general rule, the legal profession is wary of formal techniques - often finding these inefficient, confusing.[127] Any

[125] Genesereth, *Computational Law: The Cop in the Back Seat*, at 5.

[126] Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 19-20.

[127] Vern R Walker. "Discovering the Logic of Legal Reasoning". In: *Hofstra Law Review* 35 (2007), pp. 1687–1707, at 1692-1693.

additional complexity must contribute to solving actual legal problems (as well as the bottom line of its practitioners).

2.6 Appropriate Fields of Law

Computational law's philosophy contrasts with the notion of Legal Realism. In its stronger formulation, legal realism means that the text of the law doesn't matter, or at least does not matter as much as other considerations, in order to perform a balancing of interests (usually based on factors extraneous to law) on a case-by-case basis.[128] Computational law may not be able provide this kind of normative flexibility.[129] Instead, it is more closely aligned with Legal Formalism. Thus it carries the notion that laws are definitive, and exhaustively account for all the preferences and calculations of the legislator.

Given its alignment and limitation, Genesereth suggests that computational law is most relevant to civil law jurisdictions - where the text of the law are interpreted literally or with very constrained space for interpretation. In contrast, it is least relevant in common law jurisdictions marked with on-the-fly legal innovation through judicial interpretation.[130] There is support for the assertion that computational law is more suited for civil law over common law - based on civil law's reliance on top-down deductive reasoning from explicit codified rules (as opposed to the inductive and analogical reasoning based common law): Civil law systems can be considered as isomorphic with computer code (i.e. it is possible to draw a direct mapping between legal text and code) since civil law codification aspires towards comprehensiveness, systemic organization, and explicit definition of rules, parameters, and exceptions.[131] The reliance of symbolic AI-based computational law on

[128] In its extreme formulations, legal realism can go against the project of building a rules-based society. The author also has more practical objections: If we are not in the business of building and then recognizing enduring legal norms, then we are wasting our time teaching our students legal research and statutory interpretation. Better to instruct them on the non-legal mechanisms that actually shape decisions, such as economic interests and individual psychology.

[129] Genesereth, *Computational Law: The Cop in the Back Seat*, at 5.

[130] Ibid., at 5.

[131] Amedeo Santosuosso and Giulia Pinotti. "Bottleneck or Crossroad? Problems of Legal Sources Annotation and Some Theoretical Thoughts". In: *Stats* 3.3 (Sept. 9, 2020), pp. 376–395. ISSN: 2571-905X. DOI: 10.3390/stats3030024. URL: https://www.mdpi.com/2571-905X/3/3/24 (visited on 05/29/2026).

rule-based reasoning engines mirrors the judicial method of civil law jurisdictions, where a judge applies written statutory criteria directly to the facts to arrive at a deterministic conclusion.[132]This approach, based on formal deductive logic has also been successful when it comes to administrative law, tax compliance, social benefit allocation. On the other hand, the deductive law approach does not work well with common law, since the latter relies on stare decisis, which can introduce ambiguity and unclear boundaries - elements which are hard to mode through formal logic.[133]

Although computational law has limits when applied to cases that require analogical or inductive reasoning (which often characterizes the reasoning in judge-made Laws), Genesereth suggests that the judicial process itself can generate categorical constraints from vaguely worded statutes. Judicial law can be a source of encoded rules.[134] Even in common law jurisdictions, however, there are categorical, codified statutes that may not be subject to significant judicial discretion. Examples include legislation on data privacy, securities, enterprise management, construction, electronic commerce, taxation. There is a growing tendency in these fields of law to move toward greater textual specification and codification. This makes them more amenable to the computational approach.[135]

Extending from the above premises, the encoding exercise can demonstrate that competition law norms are also amenable to computational encoding. When codified as True or False threshold tests, and correlated with the economic literature which has referents to determinable concepts like price, and traceable lines of reasoning competition law can take on the same character as civil law. Thus, to the extent that competition norms can be restated in terms of economic principles, we can proceed with a more rigorous, objective basis for modelling norms as computational objects. This can also

[132]Henry Prakken. "Legal Reasoning: Computational Models". In: *International Encyclopedia of the Social & Behavioral Sciences*. Elsevier, 2015, pp. 784–791. ISBN: 978-0-08-097087-5. DOI: 10.1016/B978-0-08-097086-8.86161-9. URL: https://linkinghub.elsevier.com/retrieve/pii/B9780080970868861619 (visited on 05/29/2026).

[133]Ibid.

[134]Genesereth, *Computational Law: The Cop in the Back Seat*, at 6.

[135]To a certain extent, the end goal of the adjudicatory process is to come up with categorical interpretations of existing statutes. One can consider rules expressed in judicial decisions as expressed in judicial decisions as extensions of the legal text, and encode them computationally, as if they were part of the original statute. So to the extent that statutes are considered vague in a common law jurisdiction, judicial decisions can supplement them by coming up with interpretations which can be encoded. Genesereth is also convinced that as Computational Law becomes more useful, legislators and regulators will be encouraged to have more such categorical laws ibid., at 6.

help in many sources of ambiguity (such as overlapping semantics, cultural and emotional valence).

Chapter 3

Law as a Computable Structure

3.1 Computability and the Law

3.1.1 The Nature of Computability

The premise of computational law is that once we have both rigorous formal representations of law, and the appropriate logical methods to analyze them, law becomes computable. What is meant by a computable approach, or the computability of legal determinations? The formal meaning of a problem or a domain's computability relates to whether or not it can be solved through an algorithm. In other words, a problem is computable if there exists a step-by-step procedure that can be executed by a computer to solve the problem.[1]

Computability also means that once we have abstracted enough of the most important attributes of a thing into a formalized model - we can map its behavior backward and forwards in time. We can access powerful shortcuts to the things behavior - to diagnose, analyze, and predict.[2] The modern

[1] The notion of computability is derived from Alan Turing's description of problems that are amenable to an algorithmic solution (to be carried out by a computational model such as a Turing Machine). Alan M. Turing. "On Computable Numbers, with an Application to the Entscheidungsproblem". In: *Proceedings of the London Mathematical Society* 2.42 (1937), pp. 230–265. DOI: 10.1112/plms/s2-42.1.230.

[2] Without computability, we are confined to recording descriptions of phenomena, and we are limited in our ability to draw insights and make predictions abut a system. Similar to the state of astronomy before Newton developed the formalisms of calculus - without a proper computational model for celestial mechanics, all that could be done was observation and recording.

world we have was achieved through computation - from bridges to bombs to games and deep space probes. These are possible because we could build models of the forces of nature, and predict their interactions through logic and mathematics.

3.1.2 Can the Law be Computable?

Wolfram argues that the computability of law can flow from the computational character of nature, from which all phenomena (including humans and human institutions from which laws are derived)[3]: The universe itself is built on a computational foundation, and our current computational tools for representing and analyzing knowledge is the latest (and perhaps ultimate) in a series of formalisms for representing and understanding reality.[4] It is not necessary to get into such a fundamental claim. As will be argued below - it should be enough that the computational approach capture what is essential of legal knowledge. Law is not magic - it occurs within the same universe that is, to some extent, discoverable. A premise of the law as a practical profession and an academic field is that it is knowable, and that legal reasoning can be systematized. At a high level of abstraction legal processes can be modeled as if it were a computational process. We have an input (laws, evidence), and we expect an output (in the form of a decision). Formalization and computation promises to make that process purer: free from bias, fatigue, and ignorance.[5]

Complexity, incompleteness, and computational irreducibility Of course, modeling the forces acting on a physical system is one thing, but trying to model the behavior of people and institutions under the constraint of law is a different category. As mentioned in the previous section, complexity and incompleteness conspire against us. The open-textured nature of legal concepts like "justice" means that our representations and analytical tools can only go to certain levels of description. Even

[3] See Stephen Wolfram. *How to Think Computationally about AI, the Universe and Everything*. Stephen Wolfram Writings. Oct. 27, 2023. URL: https://writings.stephenwolfram.com/2023/10/how-to-think-computationally-about-ai-the-universe-and-everything/ (visited on 12/14/2023).

[4] See generally Stephen Wolfram. *A Project to Find the Fundamental Theory of Physics*. Champaign, Illinois: Stephen Wolfram, LLC, 2020. 770 pp. ISBN: 978-1-57955-035-6.

[5] Christopher Markou and Simon Deakin. "Ex Machina Lex: Exploring the Limits of Legal Computability". In: *Is Law Computable? Critical Perspectives on Law and Artificial Intelligence*. 2020, pp. 31–65, at 33.

if we can somehow develop a rich enough toolset to capture legal concepts, Gödel's incompleteness means that there will always be a gap in our formalization.[6]

Some problems are subject to computational irreducibility.A problem is computationally reducible if it allows some computational shortcut to determine the solution more rapidly. Even seemingly simple systems that can be defined formally may turn out to be computationally irreducible. Wolfram suggests that law may be considered as computationally irreducible. While lawyers and academics may use their insight and experience to make predictions as to the solution to legal problems.[7] However, since legal problems involve not just rules but other variables (evolving evidence, human emotions, changing laws, judicial discretion), law has the property of computational irreducibility, the only way to arrive at the solution is to proceed with the whole process of adjudication. That is, even if we can reduce a system's behavior into simple rules, it is still possible for complex behavior to arise from such systems. It may not be possible to make a prediction about a systems state or behavior past a certain point (even if the system's behavior can be modeled algorithmically). Wolfram suggests the ramification for the design of rules and legal systems - i.e. that it may not always be possible to predict in the abstract the consequences of certain rules. Instead, he suggests an empirical approach where we can see for ourselves the effects of these rules.[8]

This difficulty does not mean that the problem will be intractable. Wolfram states that in the teeth of complexity and incompleteness, even the hard sciences are beset by oceans of non-computability. Despite all their progress in theory-making and theory-testing, scientists still have to contend with a universe that largely resists mathematical certainty. And yet, they have found enough islands of computability amidst that ocean to lay the foundations of useful things like engineering, computer

[6] Gödel's theorems on the fundamental incompleteness of any axiomatic system impacts mathematics and logic, and ultimately, the capacity of computational formalism to model reality Richard P. Feynman. *Feynman Lectures on Computation*. Ed. by Anthony J. G. Hey and Robin W. Allen. Boca Raton: CRC Press, 2018. 303 pp. ISBN: 978-0-7382-0296-9, at 52.

[7] Stephen Wolfram. *A New Kind of Science*. Champaign, Illinois: Wolfram Media, 2002. 1197 pp. ISBN: 978-1-57955-008-0, at 741-745.

[8] Stephen Wolfram. "AI Law and Computational Irreducibility". FutureLaw 2023, Stanford Law School. Apr. 25, 2023. URL: https://www.youtube.com/watch?v=8oG1FidVE2o (visited on 01/14/2024), at 3:53.

science, particle physics.[9]

Our models for law will likely be incomplete and thus inaccurate. But the incompleteness of a model does not mean it will be useless. A map will never be as detailed as the territory that it guides us through, but a good map should have enough information to be useful.

A formal approach can allow smoother, more reliable collaboration and the building of higher "towers of consequences"[10] - systems that will allow more detailed study of legal systems, as well as applications for real world problems that involve the law. For example - the ability to encode legal rules into a computer program may be the key to encoding firm, normative("constitutional") limits on artificial intelligence that can still be read, understood, and edited by humans.

From natural language to formal encoding We already use a stylized version of natural language, "legalese" in order to gain some precision in laws and contracts. We can thus consider legalese as a partial formalisation of the law. However, it will still fall short in providing precision. This is because at a fundamental level, these linguistic representations rely on the meaning of words. For natural languages, meanings are defined by the practice and experience of the users of the language.[11] The "semantic drift" which characterizes natural languages used in formalizing law can be avoided (or mitigated) by using computer languages, where concepts can be defined in advance to be at a certain level of precision. The problem is that most computer languages usually don't directly deal in subjects of the real world that are important to law.Wolfram discusses the possibility of a knowledge-based computer programming language (such as Wolfram Alpha), that links to a knowledge base in order to provide unambiguous (if partial) definitions for both abstract and tangible entities.[12]

[9] See generally Wolfram, *A New Kind of Science*.

[10] Wolfram demonstrates how a field can progress through a better formalization and encoding system: Prior to the invention of algebraic notation, problems were described through natural language text (which can be imprecise). A more formal, streamlined method made it easier to share and build off each other's ideas. Wolfram, *How to Think Computationally about AI, the Universe and Everything*.

[11] Wolfram, "Computational Law, Symbolic Discourse and the AI Constitution", at 145.

[12] Ibid., at 145-146.

The main bottleneck that prevents us from formalizing human law in the same way that we have formalized physics, logic, or mathematics is that law deals with human activities. While we have a general formalism for particles and populations, we don't have a formalism that operates at the level of description required by law. According to Halper, a way around this is discarding the notion that legal reasoning should be exactly like mathematical or scientific reasoning: Scientific reasoning is descriptive (as even its prediction are a form of subsequent description).Legal reasoning, on the other hand, is prescriptive; legal statements are commands that express "the existing balance of power between competing interests in a society".Legal reasoning is not an aid to prediction but to decision making.[13] Uncertainty is often most pronounced in specific areas of the law, such as constitutional law, or when the discourse involves concepts like ownership that are actually the result of accretion of many rights and privileges being bundles into a single name. However, concepts that are not definable with rigor are not necessarily meaningless (the way they would be in science), but simply vague - certain elements may still be considered for or against its use on a case to case basis.[14]

Under this premise, a rule of law can be expressed in the form of a conditional proposition: IF (antecedent(s)) THEN (consequent).[15] Instead of *a priori* notions or empirical findings, legislation provides truth (as major premises) that will be the subject of deductive syllogism, as well as truth that will be the basis for inductive reasoning in law.[16] The same applies to the rationale and dispositive portion of decisions, since a case is pleaded through a variable statement of claim (in several causes of action) by trying to establish how the case satisfies the relevant law by: 1. Statement of how the facts of the case particularize the antecedents in the relevant rules; 2. Statement of the final consequent of these rules in terms of the claim.[17]

[13] Thomas Halper. "Logic in Judicial Reasoning". In: *Indiana Law Journal* 44.1 (1968), pp. 33–48, at 46-47.

[14] Ibid., at 47-48.

[15] Ontological situations in law might exist in an ontology of potentialities and possibilities. Pamela Gray. "The Ontology of Legal Possibilities and Legal Potentialities". In: *Proceedings of LOAIT 07*. II Workshop on Legal Ontologies and Artificial Intelligence Techniques. 2007, p. 7–23, at 8.

[16] Ibid., at 10.

[17] Ibid., at 9.

3.2 Confronting Objections to Logic in Law

Computational law requires some role for logic in legal reasoning. A significant goal of computational law is the production of a computer system capable of producing legal advice (as opposed to just textual information). This can only be possible if logic has a place in law. Because if anything, a computer system's only actual capability is demonstrating a logical system.[18] Similarly, only a logical system can be computerized.[19] Even in the long term, understanding and designing AI systems involved in legal reasoning will require a background in logic, since AI applications (even those that seemingly interact through natural language), will have programming that will be undergirded by formal logic.

Lawyers have built a conceptual moat around the field of law, to distinguish it from the hard sciences, claiming that the law, unlike these fields, will always evade a reductionist, logical approach.[20] The propositions of a legal system are not like the fixed axioms and postulates of Euclidean geometry. Rather, they are grounded on language and jurisprudence - which can be indefinite and changeable over time. The semantics of words in a statute will be subject to the influence of psychology and sociology.[21] Thus, its concepts and reasoning are not amenable to computation because these are largely not computationally reducible. Since legal concepts and rules are socially constructed and in flux, they cannot be fully represented into numbers and logical constructs.

[18] Philip Leith. "Logic, Formal Models and Legal Reasoning". In: *Jurimetrics* 24.4 (1984), pp. 334–356, at 334.

[19] Ibid., at 334.

[20] Jeffrey Goldsworthy. "The Limits of Judicial Fidelity to Law: The Coxford Lecture". In: *Canadian Journal of Law and Jurisprudence* 24.2 (July 2011), pp. 305–325. DOI: 10.1017/S084182090000518X, "The popular impression of legal thinking is that it is logically rigorous. But legal reasoning, whether of judges, advocates or legal scholars, rarely has the clarity and rigour of the best analytical philosophy. Often this is because the subject-matter is simply incapable of being treated as rigorously. But more importantly, legal reasoning in real cases leads to practical decisions that have drastic effects on individual's lives or the welfare of the community, for which judges properly feel some moral responsibility. Consequently, legal reasoning can have a tendentiousness—an almost palpable gravitation towards a desired conclusion—that is lacking in the work of analytical philosophers, pure mathematicians or nuclear physicists."

[21] Anthony G. Guest. "Logic in the Law". In: *Oxford Essays in Jurisprudence*. Oxford: Oxford University Press, 1961, pp. 176–197, at 177.

The objections in the legal literature can fall under the following categories: 1. **Historical arguments**, i.e. that the legal reasoning has developed as a discipline separate from logic; 2. **Epistemological arguments**, which rely on fundamental difference between law and logic, not only in substance but in terms of subject matter; 3. Finally, there are the **Practical arguments** that relate to the applicability of logic to real-world legal problems. We confront these objections in the following subsections:

3.2.1 Historical Convergence of Logic and Law

Law and logic during the classical period A profession as steeped in tradition and the weight of history as law may view embedding logic as an unnecessary modernist intrusion. However, the history of law is replete with examples of the convergence of logic and law. For Aristotle, law and logic were one and the same.[22] Aristotelian logic, or what we now know as classical propositional logic, was derived from analysis and systemization of legal arguments and decisions.[23] Winkler suggests that Roman legal writers likely adopted contemporary logic and mathematics for the development of fact-patterns. An example of such adoption would be in *Julian P.35,2,87*, which used Euclidean concepts (like commutativeness) to generate hypotheticals in inheritance law.[24] This was carried on through the scholastic tradition, which viewed law as a system of rules which can be logically deduced from immutable principles.[25] These principles, in turn, can be

[22] Lee Lovevinger. "An Introduction to Legal Logic". In: *Indiana Law Journal* 27.4 (Sum. 1952), pp. 471–522, at 471, citing A Treatise on Government, or The Politics of Aristotle, Book III, c. 16, Elli's translation, 1943.

[23] See Wolfram, "AI Law and Computational Irreducibility", at 14:13; See also Wolfram, "Computational Law, Symbolic Discourse and the AI Constitution", at 145. An intriguing notion propounded by Wolfram is that laws are in fact the original inspiration for logical and mathematical systems. Legal arguments served as the model for the axiomatic approach to geometry defined by Euclid. Later, in the development of scientific thought, the discovery of "natural laws" were viewed as similar to legislation, i.e. These define constraints from God (or nature) instead of a human lawmaker.

[24] See generally Markus Winkler. "The Use of Logic for Creating Fact Patterns". In: *New Developments in Legal Reasoning and Logic - From Ancient Law to Modern Legal Systems*. Cham, Switzerland: Springer Nature, 2022, pp. 41–52.

[25] See Karlheinz Hülser. "Proculus on the Meaning of OR and the Types of Disjunction". In: *Past and Present Interactions in Legal Reasoning and Logic*. Springer International Publishing, 2015, pp. 7–30, at 8. Emperor Justinian's *Digestae*, in the chapter *De verborum significatione* (On the meaning of words), contains reference to the Letters of Proculus, a distinguished Roman jurist.The passage quoted from Proculus covered his discussion on logical disjunctions (OR). The fragment from Proculus is itself derived from a long tradition of adopting concepts from Stoic logic. Through its adoption in the digests, it continues to inform modern statutory interpretation.

discovered by man through a process of reasoning. Great jurists such as Thomas Aquinas, William Blackstone also proceeded along these lines.[26] For the longest time, logic was Aristotelian logic. One of the Aristotelian logic's central theory of the judicial syllogism, where a judicial decision is justified through a form of syllogistic reasoning, i.e. as an inference from normative and factual premises.[27] This form of legal determination has arguably shaped the notion of separation of powers (i.e. between legislation and adjudication): The legislative creates law as a set of legal norms, and the judge will need to reason through these premises in order to apply them to a particular set of facts.[28]

Logic in Islamic Law An important component of Islamic law is *figh* ("understanding"), the principles of which ("usul al-figh") come in the form of logical and philosophical arguments. *Figh* is "the science of deducting binding instructions from the sources in practical forms, which are applicable for agents who are following the Divine Will". It includes a mechanism of consistent rules.[29]

Law and logic during and after the Renaissance A cornerstone of the 17th century naturalist doctrine (Grotius, Salamanca School, Espinoza) is that the principles of law should be systematized through mathematical methods. Efforts to both define and systematize characterized legal studies and there was the view that certainty of the law was attainable.[30] A crystallization of these ideas can be found in the recently rediscovered works of Gottfried Wilhelm Leibniz. Leibniz is commonly

[26] *Ibid.*

[27] Pablo E. Navarro and Jorge L. Rodríguez. *Deontic Logic and Legal Systems*. New York: Cambridge University Press, Sept. 29, 2014. ISBN: 978-0-521-76739-2. DOI: 10.1017/CBO9781139032711, at ix.

[28] Such a separation of functions assumes law has logical attributes such as: 1. Completeness - that there is always an applicable legal norm that can solve any dispute; 2. Consistency - that there are no incompatible norms applicable to the same case. Judicial decisions rely on at least one of these holding true. ibid., at ix.

[29] Islamic law has concepts that correspond to the deontic modes: 1. Obligatory action (*wajib*); 2. Forbidden action (*haram*); 3. Recommended permissible action (*mustahabh*); 4. Reprehended permissible action (*makruh*); 5. Evenly permissible action (*mubah*) Mohammad Ardeshir and Fatemeh Nabavi. "A Logical Framework for the Islamic Law". In: *New Developments in Legal Reasoning and Logic - From Ancient Law to Modern Legal Systems*. Cham, Switzerland: Springer Nature, 2022, pp. 53–82, at 54-55.

[30] Alberto Artosi and Giovanni Sartor. "Leibniz as Jurist". In: *The Oxford Handbook of Leibniz*. Ed. by Maria Rosa Antognazza. Oxford University Press, Dec. 27, 2018, pp. 640–663. ISBN: 978-0-19-974472-5. DOI: 10.1093/oxfordhb/9780199744725.013.38. URL: https://academic.oup.com/edited-volume/34667/chapter/295400716 (visited on 10/10/2023), at xviii.

known a a leading figure in mathematics and philosophy. However, before his seminal work in those fields he was a lawyer and a promising legal scholar. Leibniz embarked on a project to develop a system that unified language, logic, and mathematics into a consistent framework, free from ambiguity and capable of mechanistic, symbolic manipulation. He hoped to apply rigorous mathematical thinking to all branches of human knowledge, particularly law.[31] His work combines law and philosophy, and proceeds from the premise that some of law's fundamental questions cannot be answered without philosophical thought.[32] Leibniz insisted that law should have a "philosophical basis", without which the law is bound to be an "inextricable labyrinth".[33] His forays into philosophy and law seems to be partially motivated by his numerous attempts at reconciling church doctrines (Protestants v. Catholics), conflicts over which led to the Thirty Years war that destroyed Germany. His works on legal reasoning have only been translated and published recently, indicating that he pursued a mathematical-logic approach similar to modern ideas in computational law.[34] Leibniz's first legal dissertation, *Disputatio juridica de condetionibus* used propositional logic, modal logic, and probability logic to the law on conditions, a technical problem under Roman law.[35] His writings indicate that this direction was inspired by classical sources, which requires that law, as the "science of the just and unjust", be built on "the awareness of human and divine affairs".[36] Leibniz's interest in Roman Law as the basis of a rational legal system is the view (shared by other jurists) that the Roman law tradition is more accepting of the convergence between law and science. Roman law is said to take into account "the working of nature" in order to produce sound and equitable decisions.[37]

[31] Deakin and Markou, "From Rule of Law to Legal Singularity", at 5.

[32] Matthias Armgardt. "Leibniz as a Legal Scholar". In: *Fundamina* (2014), pp. 27–38, at 28-29, citing *Specimen quaestionum philsophicarum ex jure collectarum, 1664*.

[33] Note that Leibniz was refering to Philosophy in its broader, classical sense, which includes logic and mathematics Artosi and Sartor, "Leibniz as Jurist", at xx.

[34] Leibniz'a view on legal certainty rests partly on similarities between geometry and jurisprudence: "Both have elements and both have cases. The elements are simples (simplicia); in geometry figures, a triangle, circle, etc; In jurisprudence an action, a promise, a sale, etc. Cases are complexions (complexiones) of these, which are infinitely variable in either field." ibid., at xxv.

[35] Armgardt, "Leibniz as a Legal Scholar".

[36] Artosi and Sartor, "Leibniz as Jurist", at 5 citing Ulpian, D.1.1.10.2, *De justitia et jure*.

[37] Ibid., at 6.

The three underlying ideas of Leibniz's legal investigations are:[38]

1. Legal research and problem solving, particularly adjudication requires an interdisciplinary dialogue. The law needs to accept ideas from other disciplines such as philosophy, logic, theology, mathematics, and physics.

2. Law also needs to have an intradisciplinary dialogue, i.e., between the various schools of legal thinking.

3. Law requires a more diverse range of reasoning methods and cognitive tools. Practitioners can select the appropriate tool based on pragmatism, i.e. their effectivity in solving legal problems.

Leibniz believed that no case, no matter how apparently perplexing, is insoluble *ex jure*.[39] Thus, he applied logic to confront legal puzzles from the classical era, arriving at a classification scheme for apparent and actual legal conundrums and the appropriate analytical device to solve them:[40]

1. Cases of apparent conflict between law and philosophy (which during that time included metaphysics, mathematics, empirical sciences, theology) that often arise from the same terms (but with different meanings) used in law and philosophy.

2. Questions that arise from the assumption that a principle is of universal application, but is in fact justifiable under particular pragmatic conditions, or simply the result of defects in the underlying conceptual frameworks used by lawyers and jurists.

3. Problems that arise from the lack of a deeper logical analysis of a conceptual issue.

4. Actual legal puzzles which are cases of doubtful solution because of the convoluted form of dispositions (expressions of intent), or conflict with a priority relationship.

[38] Armgardt, "Leibniz as a Legal Scholar", at 5.

[39] Leibniz argued, based on rationalism, that every legal question can have only one answer. Gottfried Wilheim Leibniz. "Dissertatio de Arte Combinatoria". Leipzig: University of Leipzig, 1666. URL: https://www.math.ucla.edu/~pak/hidden/papers/Quotes/Leibniz-Arte-Combinatoria.pdf.

[40] This position also made him wary of judicial discretion Artosi and Sartor, "Leibniz as Jurist", at ix, xxi.

Leibniz himself acknowledged that reasoning through legal problems will require more than propositional logic, since such problems involve uncertainty, possibility, and the passage of time. Although Leibniz's efforts to develop a logical formalism was not successful, these ideas, inspired systems of logic and continue to animate the field of computational law.[41]

It was the notion of the rationality of law that partly animated the law codification movements, which in turn makes the law more accessible and practical.[42]

The challenge of legal realism The controversy is best represented by the divergence between Christopher Columbus Langdell (who sought to analyze/organize contract law in a systematic, scientific fashion) and Justice Oliver Wendell Holmes, who criticized this approach. Langdell pursued, as the basis of modern legal instruction in the United States, the axiomatization of law, differing with Leibniz as to the source of such axioms. For Leibniz axioms should be derived from a variety from a variety of domestic statutes and natural law. For Langdell, the axioms should be derived from the mass of decisions (in the same way that scientists derive axioms from individual observations).[43]The most potent historical challenge to the notion of identity between logic and law comes from Justice Oliver Wendell Holmes Jr.[44] Legal scholars continue to cite this epigram as an embodiment of the school of legal realism: "The life of the law has not been logic, it has been experience".[45]

Courts and advocates in the Philippines have cited this quote from Holmes, often without its full context to the point that it has become a slogan, or the legal equivalent of a meme. It can be invoked to defeat a clear interpretation of the law on linguistic and rational grounds in order to introduce

[41] Ibid., at 11.

[42] Dean Alfange Jr. "Jeremy Bentham and the Codification of Law". In: *Cornell Law Review* 55.1 (Nov. 1969).

[43] Deakin and Markou, "From Rule of Law to Legal Singularity", at 12.

[44] See Lovevinger, "An Introduction to Legal Logic", at 472.

[45] Holmes, The Common Law, p. 1, 1881.The full quotation is as follows:"The life of the law has not been logic: it has been experience. The felt necessities of the time, the prevalent moral and political theories, intuitions of public policy, avowed or unconscious, even the prejudices which judges share with their fellow-men, have had a good deal more to do than the syllogism in determining the rules by which men should be governed. The law embodies the story of a nation's development through many centuries, and it cannot be dealt with as if it contained only the axioms and corollaries of a book of mathematics..."

extraneous considerations. However, the reflexive invocation of this epigram in order to frustrate the application of logic is misleading.

If one were to read the rest of Holmes' work, one would realize that Holmes was not dismissing the role of logic and rational thinking in law. Instead, Holmes was urging us to include more inputs into what is still a logical process of making a legal determination.

In objecting to what he called "the fallacy of the logical form", Holmes:

1. Acknowledges that as a phenomena contained in the same universe as physical matter, law is ultimately subject to the same underlying rules, such as causation (otherwise, it would be a miracle);

2. Acknowledges that logic permeates through the practice: "The training of lawyers is a training in logic" - since it involves building familiarity with logical tools like analogy, discrimination, and deduction. Holmes also characterizes judicial decision as expressed in the language of logic.

Thus, Holmes objection, and the actual divide between "natural law" and legal realism is not whether or not logic should be applied at all, but to what materials logical processes should work with. For the "natural law" school, they believe that there are transcendent basic principles which can be grasped intuitively, or derived through deduction. On the other hand, "legal realists" reject *a priori* transcendent rules and emphasize an inductive approach from empirical data (or experience).

Hawkins, through a historical and textual analysis, argues that the statement was never meant as a practical guide for legal reasoning, or the interpretation of constitutional or statutory law. Instead, it is a descriptive view of the development of the common law. The "logic" that the statement describes as being eschewed by the common law tradition is not logic as academically understood or colloquially known, but refers to the "vain attempt to impose consistency on intuitively

developed law".[46] To the extent that Holmes's words can serve as a foil to the application of logic in law, Hawkins finds that there is ambiguity as to the scope of his objections, and thus its actual application in a legal: Is it that logical reasoning has no place in law - that lawyers and judges should embrace irrationalism or intuition? Or perhaps, more realistically - was Holmes merely asking for a counterweight against excessive legal formalism?[47] Furthermore, if "experience" defines the content of the law - what constitutes this experience. More pointedly - whose experience matters?

Properly contextualized, Holmes quotation was a reaction to Landell's conception of a legal system, which was that at the core of a legal system, one can find a collection of interconnected concepts that relate to each other in configurations of doctrines and principles.The proper isolation, recognition, of these concepts would allow us to analyze and classify the growing base of rules and decisions (or resolve incompatibilities).[48] Langdell was also concerned about the place of law in academics and the larger enterprise of science. Lacking in formalization, then it would be just any "species of handicraft" best learned through apprenticeship, as in a trade.[49]

Holmes criticism against Langdell's formalism and logic was that it ignored "forces outside the law" - the prevailing moral and political thinking, considerations of policy and convenience, even prejudice. Nevertheless, Holmes acknowledged that law was a logical development - in the sense that it is subject to a reasonable account.The procedures and language adopted by legal professionals is logical in terms of form, if not substance, proceeds along logical lines - using tools such as deduction, distinction among classes, and deduction.[50] Holmes never denied the importance of legal logic, and in fact not only emphasized core ideas (like identity), but provided many instances of

[46] See generally Brian Hawkins. "The Life of the Law: What Holmes Meant". In: *Whittier Law Review* 33 (Winter Issue 2012), pp. 323–370.

[47] Ibid., at 325.

[48] Susan Haack. "On Logic in the Law: "Something, but Not All"". In: *Ratio Juris* 20.1 (2007), pp. 1–31. ISSN: 0952-1917, 1467-9337. DOI: 10.1111/j.1467-9337.2007.00330.x. URL: https://onlinelibrary.wiley.com/doi/10.1111/j.1467-9337.2007.00330.x (visited on 03/26/2024), at 3.

[49] Ibid., at 6.

[50] Ibid., at 4-5.

hard legal problems solved by logical analysis.[51]

Both Langdell and Holmes (and even modern legal practitioners) had a limited view of logic and its capabilities. For them, logic meant Aristotelian syllogisms. They were unaware of the advances from Gottlob Frege and Charles Sanders, that supersede Aristotelian syllogistic logic.[52]

It should also be noted further that logic has evolved from Holmes' schoolboy days, when most likely education would only cover classical propositional logic (or syllogistic logic as originally systematized by Aristotle)It can be conceded that classical propositional logic, as formulated during Holmes' time, the logic that most of us are aware of (and the one usually employed in programming) is not the most appropriate tool for representing legal rules. Subsequent sections will discuss the more appropriate logical systems for representing legal rules, such as deontic logic and defeasible logic.

3.2.2 Epistemiological Divide Between Law and Logic

Objections to logic often point to a fundamental difference not just in method (structured, formal versus discursive and intuitive), but also to their subjects. The basis of logic was the assumption that valid argument can be based upon the elemental form of the proposition, composed of a subject and a predicate linked by a connective. Any proposition, meanwhile has a truth value - either it is true or false. There is nothing in between (the law of the excluded middle).[53] On the other hand, legal propositions are normative rather than fact-stating, and we only have an incomplete picture of the general logic of norms.[54]

[51] Morris R. Cohen. "The Place of Logic in the Law". In: *Harvard Law Review* 29.6 (Apr. 1916), p. 622. ISSN: 0017811X. DOI: 10.2307/1326498. JSTOR: 1326498. URL: https://www.jstor.org/stable/1326498?origin=crossref (visited on 08/15/2020).

[52] Haack, "On Logic in the Law", at 3.

[53] Leith, "Logic, Formal Models and Legal Reasoning", at 336.

[54] Robert S. Summers. "Logic in the Law". In: *Cornell Law Faculty Publications* (Paper 1133 1963), pp. 254–258. URL: http://scholarship.law.cornell.edu/facpub/1133, at 254.

In a practical legal setting (such as a case in litigation) not all premises are true, and and logic can do very little to determine their authenticity.[55] Traditional logicians are used to the deductive logic of mathematics, in which proof begins with axioms and deduces conclusions downward. They may therefore overlook this fundamentally different orientation of legal reasoning.[56] In contrast, the rule-based deductions of legal reasoning do not rest upon self-evident axioms, but instead upon the truth or falsehood of the relevant issues of fact, which are defined by rules adopted by legal authorities. The soundness of the reasoning depends upon the appropriateness of the rules governing the reasoning and the plausibility of the relevant evidence. Another feature of rule-based legal reasoning that challenges traditional deductive logic: the possibility of changing the rules themselves as a result of the reasoning. For example, in common law systems, courts can elaborate new legal rules that apply not just to the case at hand, but all similar cases in the future.[57]

3.2.3 Rationality and Consistency in Law

Legal reasoning requires coherence We evaluate law based on its coherence and its adherence to principles. However, the language of social convenience, put forward by the legal realism school, may not be sufficient to sustain a coherent body of law.[58] Logic can supplement natural language rules toward better coherence and consistency.[59] Threading through various systems of law (across jurisdiction and time periods) is the requirement that legal norms or decisions under legal authority be "reasonable." What is reasonable has been thought to be identical, or at least overlap with rationality. While reasonableness has been associated with broader considerations (such as "values"), rationality is often closely associated with logic, efficient nexus between means and ends, and testing

[55] Furthermore, it is likely that legal functionaries will be interested in issues other than the truth of propositions, as well as their logical structure. Ruggero Aldisert, Stephen Clowney, and Jeremy Peterson. "Logic for Law Students: How to Think Like a Lawyer". In: *University of Pittsburgh Law Review* 69 (2007), pp. 1–22, at 21.

[56] Walker, "Discovering the Logic of Legal Reasoning", at 1695-96.

[57] Ibid., at 1695-96.

[58] Guest, "Logic in the Law", at 196.

[59] Geoffrey Samuel. "Is Legal Reasoning like Medical Reasoning?" In: *Legal Studies* 35.2 (2015), pp. 323–347. ISSN: 0261-3875, 1748-121X. DOI: 10.1111/lest.12063. URL: https://www.cambridge.org/core/product/identifier/S026138750000581X/type/journal_article (visited on 12/18/2023), at 335.

for the reliability of beliefs.[60] Lashbrooke is of the view that law, while ultimately cannot have the same kind of immutable, universal rules discoverable such as those in the hard sciences, it is nevertheless quasi-coherent in terms of a persistent degree of the organization of legal knowledge. Although at one level,the law engages common sense and is aligned with human goals - which is harder for algorithms to perfectly replicate. On the other level, the law is concerned with justification of legal decisions in logical terms - this can be the province of algorithms.[61] Walker asserts that logic (albeit not in its classical form) is applicable throughout the legal enterprise: from deciding of cases (including weighing evidence and interpreting law), to the formulation of new policy.[62] Although the law does not have mathematical models - it has persistent institutional models, which provide a basic framework for viewing reality that includes: 1. more or less coherent definitions for persons, things, remedies, and their relationships with each other; 2. principles that contain lines of inquiry and interpretation to plausible lines of reasoning.[63]

Legal systematization around logic Law has to be rational to a certain degree, otherwise it cannot fulfill its function under constitutional political systems, which requires: 1. Decisions (whether for policy or adjudication) that are consistent with facts; 2. That those facts determined with some degree of accuracy and efficiency.[64]. Schemes for analysis and classification is also necessary for the law is to maintain continuity and usability.[65] Rationality need not be confined to a single mode of thinking but can be treated as a pragmatic tool for meeting specific ends - consistency logicians, or efficiency for economists. At its core minimum, it only requires a commitment to some process,

[60] Robert Alexy. "The Reasonableness of Law". In: *Reasonableness and Law*. Dordrecht: Springer Netherlands, 2009, pp. 5–15, at 5.

[61] E C Lashbrooke. "Legal Reasoning and Artificial Intelligence". In: *Loyola Law Review* 34.2 (1988), pp. 287–310, at 287.

[62] Walker, "Discovering the Logic of Legal Reasoning".

[63] Samuel, "Is Legal Reasoning like Medical Reasoning?", at 328; Also Walker, "Discovering the Logic of Legal Reasoning", at 1702.

[64] Ronald J. Allen. "Taming Complexity: Rationality, the Law of Evidence and the Nature of the Legal System". In: *Law, Probability and Risk* 12.2 (2013), pp. 99–113. ISSN: 1470-8396, 1470-840X. DOI: 10.1093/lpr/mgs008. URL: https://academic.oup.com/lpr/article-lookup/doi/10.1093/lpr/mgs008 (visited on 10/20/2023), at 100.

[65] Edward W. Cleary. "Presuming and Pleading: An Essay on Juristic Immaturity". In: *Stanford Law Review* 12.1 (Dec. 1959), pp. 5–28. ISSN: 00389765. DOI: 10.2307/1226735. JSTOR: 1226735. URL: https://www.jstor.org/stable/1226735?origin=crossref (visited on 12/11/2024), at 7.

subjecting assertions to operations such as deduction, induction.[66] The use of logic as the framework for a systematization of the law is not just academic pedantry but provides a useful service. Just as abstract mathematics provides tools to solve physics problems, abstractions in law can give us the ability to handle diverse set of legal situations.[67] At the very least, the appurtenances of logic (such as its terse notation, as well as its processes) can allow the lawyers to check their own assertions from first principles,just as scientists can use logic and mathematics to revise their theories based on new evidence.[68]

3.2.4 Misapprehension of "Logic"

Synthesizing the arguments of A.G. Guest and other legal philosophers, Summers argues that most objections to a more logical approach to law is often based on a misuse of the concept of "logic" itself. Upon closer inspection, even basic logical propositions do not refer to things in nature, but concepts that may not necessarily be subject to true-or-false evaluation. [69]Summers adds that most likely, these statements are criticisms of the reasoning in particular cases, rather than general arguments against the use of logic in legal reasoning. [70] The abuses of logic in the law cited are often the result of limitations in Aristotelian logic - a logic of subsumption better suited to other systems of knowledge.[71] More directly, the objection can be met by referring to legal pluralism, i.e. the notion that there are other forms of logic that can be used to represent legal reasoning.[72] This includes, as will be discussed below, deontic and defeasible logics.

One problem when we discuss the role of logic in law, is what we mean by logic in the first place - is it the technical, formal sense or are we using logic in the everyday, colloquial sense? Logic

[66]Allen, "Taming Complexity", at 100.
[67]Cohen, "The Place of Logic in the Law", at 625.
[68]Ibid., at 625-626.
[69]*Ibid.*
[70]*Ibid.* In cases where a decision is criticized for an "abuse of logic" (e.g. Whiteley v. Chapel), what may be at fault is the choice of legal premises, and not the (logical) manner in which the judge proceeds from premise to conclusion. Or, more often enough, it may be a problem with semantics.
[71]Cohen, "The Place of Logic in the Law", at 636.
[72]Leith, "Logic, Formal Models and Legal Reasoning", at 340.

in its formal sense relates to whether or not an argument's conclusions follows necessarily from the premises. The latter, "everyday logic", on the other hand, is concerned with whether any legal conclusion "makes sense" based on some informal standard.[73] These senses of the word "logic" are not related to each other. The main purpose of formal logic is to surface "possible forms of argument and conditions of valid argument".[74] On the other hand, everyday logic is prescriptive i.e., it involves the application of beliefs, (often grounded in social processes) as to what ought to be.[75]

Moreover, Halper points out that complaints directed towards logic in judicial reasoning is often actually directed to something other than logic, such as:

1. **Belligerent precisionism** - This happens when the court takes a shortcut by interpreting a word too literally, ignoring its context, history, and the purpose of the rule.

2. **Bad faith** - It may also be the case that the court is simply being disingenuous in order to pervert the law. The use of a seeming use of syllogisms and faulty inferences, however, does not make the bad faith logical.

3. **Misapprehension of scope** - By "logic" critics may mean the simplistic notion that a few express (or otherwise deducible) rules should apply to all situations; When in actuality the rule does not encompass the situation, but is nevertheless characterized as an inconsistency of reasoning.

4. **Maintenance of contradiction** - It may be possible that the Court is accommodating contradictory rules when it upholds a new line of reasoning while allowing a previous case to remain valid.

5. **Simplistic, rote reasoning** - The Court may just be stuck in simplistic, rote reasoning in order to avoid, or exculpate itself from moral or social considerations. And "logic" is equated with

[73] Leith, "Logic, Formal Models and Legal Reasoning", at 335-336.

[74] Ibid., at 337-338.Citing McCormick (1982), The Nature of Legal Reasoning: A Brief Reply to Dr. Wilson, Legal Studies, vol. 2, no. 3 286(1982).

[75] Ibid., at 336.

this mechanism, operationalizing the fiction of the detached judiciary.[76]

Thus, criticisms against logic in the law may be unfair considering the smaller scope of actual logic may have in legal reasoning: Logic does not determine the content of the premises upon which it operates. It does not presume to build a self-contained system free of contradictions. Nor can it cannot verify the truth or falsity of propositions. Logic is simply about whether or not deductions or inferences (based on given premises are correct). The choice and quality of such premises are outside of the scope of logic.[77]

The controversy is best represented by the divergence between Christopher Columbus Langdell (who sought to analyze/organize contract law in a systematic, scientific fashion) and Justice Oliver Wendell Holmes, who criticized this approach.

Both Langdell and Holmes (and even modern legal practitioners) had a limited view of logic and its capabilities. For them, logic meant Aristotelian syllogisms. They were unaware of the advances from Gottlob Frege and Charles Sanders, that supersede Aristotelian syllogistic logic.

Neither does formal logic need to subsume the whole concept of rationality in law. Although law utilizes deduction, induction, and analogy, it also has space for intuition, emotion, and even prejudice.[78] This integration can implement the "centaur configuration" for legal reasoning with the help of a computer - The user/operator can take care of the humanistic, creative aspects (definitions, selection of premises), while the computer can perform the logical non-creative parts.

[76] Halper, "Logic in Judicial Reasoning", at 33-35.

[77] Guest, "Logic in the Law", at 178.

[78] Guest, "Logic in the Law", at 182. See also Cohen, "The Place of Logic in the Law", Cohen urges a place for both logic and intuition, balancing each other out."A logical science of law can help us digest our legal material but we must get our food before we digest it. The law draws its sap from feelings of justice and social need."

3.2.5 The Problem of Analogy in Law

Analogy is often used in legal reasoning (especially in adjudicatory cases). The mode of reasoning involves connecting particulars, i.e. making a determination if one particular fact or set of facts is similar to another.[79] For Aldisert, it is a form of inductive reasoning which involves drawing similarities between apparently different things, i.e., new legal issues to existing established precedents: After establishing similarities between two cases, a court can announce the rule of embedded in the first case, and then apply that rule to the second case.[80] The use of analogy allows for decisionmakers to deal with probabilities, as well as bridging to unknown situations with known rules.[81]

Although analogy is often framed as a component of formal logic - it is not totally susceptible to a complete enclosure by logic. Analogy is often relied on precisely because it is not always possible (or desirable) to arrive at the correct legal answer through logical deduction or any deterministic process.[82] Reasoning by analogy is still rules-based, although the reasoning is provided piece-meal,evolving as new decisions are issued, and often implicitly.[83] Although still subject to generalized rules (i.e. connection based on similarity or fit), the scope and content of analog can be open textured.This is because the crucial step in analogy is not determining the degree or count of similarities and distinctions, but in identifying the principle from which the claim of similarity or distinction can be derived. Identification of a principle requires evaluation (not merely finding or counting textual similarities).[84] There are no set rules for what qualifies as an appropriate connecting principle between cases, as there are many ways to assess the degree to which the comparable

[79] Samuel, "Is Legal Reasoning like Medical Reasoning?", at 332; Aristotle characterized analogy as "Neither like reasoning from part to whole, nor like reasoning from whole to part, but rather reasoning from part to part, when both particulars are subordinate to the same term, and one of them is known" Guest, "Logic in the Law", at 191. Citing Aristotle, *Analytica Priora* (Ross's ed), 23, 69a, 13.

[80] Aldisert, Clowney, and Peterson, "Logic for Law Students: How to Think Like a Lawyer", at 16.

[81] Halper, "Logic in Judicial Reasoning", at 45.

[82] Ashley et al., "Legal Reasoning and Artificial Intelligence: How Computers "Think" Like Lawyers", at 254.

[83] Kowalski and Sergot, "The Use of Logical Models in Legal Problem Solving", at 216.

[84] Cass R. Sunstein. "Of Artificial Intelligence and Legal Reasoning". In: *University of Chicago Law School Roundtable,* 8.1 (2001), pp. 29–36, at 32.

attributes resemble or differ from each other in relevant aspects.[85] How can a judge or lawyer select the appropriate principle? Not just through "factual closeness" but through another principle for determining such relevance or closeness, or evaluating which principle is better. Usually this is based on social norms, empirical or policy preferences. Another approach to selecting principles is to determine whether it is normatively consistent with other beliefs. This relies on Dworkin's characterization of legal reasoning as an attempt "to make best constructive sense of past legal events" The appropriate legal analogy generates the best justifying principle out of past decisions, and applies that principle to the case at hand.[86] Since the search space for connecting principles can be vast, or new principles can be created *ad hoc*, it may not be possible to encode all these in advance. Even large language models, based as they are on a pre-existing corpus of work, may only approximate existing principles to make analogical connections. Without either ground truth based on empirical observations, or a sense of normative evaluations[87], it also cannot generate new principles.

3.2.6 Incompleteness of Formal Systems

Another aspect of the divide between law and logic is related to the necessary incompleteness of formal systems. The incompleteness of formal systems is a result of Gödel's incompleteness theorems, which states that any non-trivial formal system will contain statements that are true but cannot be proven within the system. This suggests that legal systems, as complex formal systems, might also have inherent limitations. There could be legal questions or scenarios that cannot be fully resolved within the existing legal framework. This means that there will always be gaps in any formalization of the law, and that there will always be legal questions that cannot be answered

[85] Aldisert, Clowney, and Peterson, "Logic for Law Students: How to Think Like a Lawyer", at 18.

[86] Sunstein, "Of Artificial Intelligence and Legal Reasoning", at 33. Dworkin's Law as integrity rejects the positivist view that law consists merely of explicit rules enacted by legislatures. Legal reasoning is an interpretive enterprise where judges must find the answer that best "fits" the institutional history and "justifies" it in terms of political morality. Alexander Green. "Expanding Law's Empire: Interpretivism, Morality and the Value of Legality". In: *European Journal of Legal Studies* 4.1 (2011). URL: https://ejls.eui.eu/wp-content/uploads/sites/32/pdfs/Spring_Summer2011/.

[87] See discussion in Chapter 9 below

through logical deduction.[88] This is a significant challenge to the idea of computational law, as it suggests that there will always be limits to what we can achieve through logical analysis. Without an overall general model for the world, representations in a formalism will always be incomplete.

Wolfram asserts, however that such an exhaustive scheme is not necessary, and that it would be possible to capture concepts as needed.[89] The vagueness of legal concepts, the indeterminacy (at least from a formalized, reductive standpoint) of the law is not an error of design but an important feature - one which facilitates its widespread adaptation and continued applicability.[90]

Another way forward is to accept that while logic may not fully encompass the entirety of the law's substance, but to accept that we can formalize only certain sections of the law. Surden suggests focusing on rules that are more amenable to encoding - through a property called its representational complexity. It is a measure of the degree to which issues of legal theory can inhibit computational modelling of the law as it operates in practice.[91] For example, Genesereth suggests that computational law is most relevant to civil law jurisdictions - where the text of the law are interpreted literally or with very constrained space for interpretation. In contrast, it is least relevant in common law jurisdictions marked with on-the-fly legal innovation through judicial interpretation.[92] However, the computational law approach can still be used by common law (and hybrid) jurisdictions. Even in common law countries, there are categorical, codified statutes that may not be subject to significant judicial discretion. Examples could include: data privacy, securities, enterprise management, construction, electronic commerce, taxation. The tendency of these fields

[88] Rebecca Goldstein. *Incompleteness: The Proof and Paradox of Kurt Godel*. New York, London: Atlas Books, 2005.

[89] See Wolfram, "Computational Law, Symbolic Discourse and the AI Constitution", "At a foundational level, computational irreducibility implies that there will always be new concepts that could be introduced...[C]omputational irreducibility implies that none of them can ever be ultimately be complete".

[90] Lovevinger, "An Introduction to Legal Logic", at 470.

[91] Harry Surden, Michael Genesereth, and Bret Logu. "Representational Complexity in Law". In: *Proceedings of the 11th International Conference on Artificial Intelligence and Law*. ICAIL07: 11th International Conference on Artificial Intelligence and Law. Stanford California: ACM, June 4, 2007, pp. 193–194. ISBN: 978-1-59593-680-6. DOI: 10.1145/1276318.1276355. URL: https://dl.acm.org/doi/10.1145/1276318.1276355 (visited on 08/14/2023), at 193.

[92] Genesereth, *What Is Computational Law?*, at 5; To the extent that civil law already shares this tendency to structure, practicality and rationality, it will be easier to compute compared to common law, which is more complex. Schrepel and Groza, *Computational Antitrust Worldwide: Fourth Cross-Agency Report*, at 3.

of law is for greater textual clarity, especially for its users. This makes them more amenable to the computational approach. Genesereth is also convinced that as the technology becomes more useful, legislators and regulators will be encouraged to have more such categorical laws.[93] To a certain extent, the end goal of the adjudicatory process is to come up with categorical interpretations of existing statutes. One can consider rules expressed in judicial decisions as expressed in judicial decisions as extensions of the legal text, and encode them computationally, as if they were part of the original statute. So to the extent that statutes are considered vague in a common law jurisdiction, judicial decisions can supplement them by coming up with interpretations which can be encoded.[94]

3.2.7 Practicality of Employing Logic

Other objections to accommodating logic emerge from the notion that the law is not a matter of logical necessity, but really of practicality (and of social costs and benefits).[95] Legal reasoning is pragmatic (rather than formal or mathematical) : It is oriented towards actions (or preventing) actions. While it tries to arrive at truth based on reasonable inferences, this is balanced with fairness, efficiency, mandate of a political body applying it. All this occurs within a context of limited time and incomplete information.[96]

Logic in the adjudicatory process A consequence of this argument is that logic has no use for judges and lawyers, since their conclusion are arrived at intuitively, with the reasoning is arrived at *post facto*.[97] The rule deduction skeptics adopt the position that legal decisions do not arise from deduction from existing legal rules. The legal principles that supposedly guide legal reasoning are too vague and subject to so much discretion, that the operation of logical processes is not possible.[98]

[93] Genesereth, *Computational Law: The Cop in the Back Seat*, at 6.

[94] Although computational law has limits when applied to cases that require analogical or inductive reasoning, Genesereth suggests that the judicial process itself can generate categorical constraints from vaguely worded statutes. Judicial law can be a source of encoded rules. ibid., at 6.

[95] Cohen, "The Place of Logic in the Law", at 632-633.

[96] Walker, "Discovering the Logic of Legal Reasoning", at 1691.

[97] It is asserted that the formalized, logical form of the decision is used to legitimize a decision based on emotion, prejudice, or rote of training. Halper, "Logic in Judicial Reasoning", at 36-38.

[98] Ibid., at 36-38.

To this, Halper points to fields of law, (such as real property law) that are devoid of any emotional or intuitive notions, through which systematic generalizations can be derived.[99] The non-logical intuition may thus be based on a judge being so steeped in the deeper overall logic of the law, and it only seems intuitive since he reaches his conclusions first and then justifies them later. Logic may also play a role in how a judge evaluates a particular proposition (whether or not such proposition was arrived at logically or intuitively), in that the judge reasons through the actual application of the proposition and considers its implications. Logical deduction is useful in this stage since it involves determining the effect of a proposition on the existing structure of the law. This is often conceived in logical terms, i.e. whether or not there are inconsistencies.[100] There is also the argument that we should not privilege the default ways of thinking in the law. These intuitive, psychological processes are exactly the kind we need to scrutinize with logic for possible inconsistencies. While Halper concedes that legal decision making is not purely logical, and the presence of a clear body of rules will not remove judicial discretion, or eliminate the influence of nonlegal considerations.[101]

Although not couched in formalisms of modern symbolic logic, instances of both deductive and deductive thinking are inherent in legal reasoning:

> In his selection of competing propositions and in his consideration of the propriety of subsuming a particular case under a certain general rule, a judge is not, of course, guided by logic. He is guided by insight and experience. But in his application of the proposition selected, and in his testing of its implications before he adopts it, he uses a deductive form of reasoning in order to discover its potentialities. The directive force of the principle may be exercised along the line of logical progression, and a judge must always keep in mind the effect which his decision will have on the general structure of the law.[102]

[99] Summers, "Logic in the Law", at 255. This is not to say that parties involved in a property dispute will not be emotionally motivated. Rather, this is a statement on the content of the rules that may determine and govern the issues.
[100] Halper, "Logic in Judicial Reasoning", at 36-38.
[101] Ibid., at 36-38.
[102] Guest, "Logic in the Law", at 188.

Summers criticizes that this is incomplete, i.e., that logic can play a role even in the selection of premises necessary to decide particular cases. Guest also asserts that inductive logic is not applicable to law. However, Summers points out that when lawyers advise clients they often use a form of inductive logic when they make predictions and generalizations from individual cases.[103] In a way, a lawyer already treats legal questions as a computational problem, having his own estimation function based on past data such as the history of the controversy, the applicable law, and the court's past decisions.

Logic against "Judicial subterfuge" Guest maintains that logically phrased reasoning in decisions are just abstractions of a deeper process, involving many factors, that a judge intuitively arrives at and justifies with reasoning later.[104] There is often a perceived tension between "rule of law" defined as "strict adherence to legal norms and their logical implications", and the aspiration to "do justice", often in the form of providing a "happy ending" for the individuals before them. This leads to judicial subterfuge, in the form of spurious interpretation of the law.[105] Goldsworthy acknowledges that there are hard cases characterized by indeterminate law. In which case judges must exercise creativity and in effect create new law. The problem lies in judges allowing considerations outside of the law in order to supplant determinate law. Often, the first step to this is engaging in the pretense that an otherwise determinate law is indeterminate, and thus the appropriate opportunity for deploying judicial creativity.Courts can delude themselves as to the content of the law, based on their long immersion in legal culture - which results in *post facto* legal rationalization of their intuitive convictions as to the proper legal solution. There is no evidence that a judge's intuitions as to practical consequences should be privileged over sound legal reasoning, and the preference for intuitive solutions, while appealing for the immediate case may erode the rule of law over the long term. The use of logic can provide a practical constraint on legal interpretation, bolstering it against

[103] Summers, "Logic in the Law", at 255-256.

[104] Guest, "Logic in the Law", at 187.

[105] Goldsworthy, "The Limits of Judicial Fidelity to Law: The Coxford Lecture", at 307; See also Pound's less generous characterization of spurious interpretation as an act that "puts a meaning into the text as a juggler puts coins, or what not, into a dummy's hair, to be pulled forth presently with an air discovery" Roscoe Pound. "Spurious Interpretation". In: *Columbia Law Review* 7.6 (June 1907), p. 379. ISSN: 00101958. DOI: 10.2307/1109940. JSTOR: 1109940. URL: https://www.jstor.org/stable/1109940?origin=crossref (visited on 04/28/2024), at 382.

judicial subterfuge.[106]

3.2.8 Logic Implicit in the Practice of Law

Legal reasoning is a composite of deductive logic, inductive generalization, and analogical comparison - operating within a framework of institutional constraints. A core mechanism of logic in law is the deductive syllogism - the legal rule provides the major premise, the facts the minor premise, and from the combination of these two should arise a valid conclusion.[107] For example:

1. **Major premise** - All contracts that unreasonably restrain trades are void;
2. **Minor premise** - Contract *x* unreasonable restrains trade;
3. **Conclusion** - Contract *x* is void.

There are tools used by lawyers and judges in their reasoning that are logical or at least closely related to logic (in the sense of formalized, structured argumentation):[108]

1. **Analogy** - A form of inductive reasoning which involves which involves drawing similarities between apparently different things.[109]
2. **Reductio ad absurdum** - A principle based on the logical theorem: "If a proposition implies its own contradictory it must be false". Often used in legal reasoning but can lead to overgeneralization and failure to make important distinctions.[110]

[106] Logic can also help prevent a related shortcoming of the judicial process, that of "well-meaning sloppiness of thougt" - characterized by undefined or poorly defined concepts, failing to interrogate the rigor of arguments Goldsworthy, "The Limits of Judicial Fidelity to Law: The Coxford Lecture", at 318.

[107] Deductive logic is the substance of most judicial decisions, i.e. when the conclusion is compelled by known facts - If A and B are true, then C must also be true. Aldisert, Clowney, and Peterson, "Logic for Law Students: How to Think Like a Lawyer", at 3; Samuel, "Is Legal Reasoning like Medical Reasoning?", at 333.

[108] Lovevinger, "An Introduction to Legal Logic", at 478.

[109] Analogy usually proceeds through the following process: 1. Establishing similarities between two cases;2. Announcing the rule of law embedded in the first case; 3. Apply the rule of law to the second case. Aldisert, Clowney, and Peterson, "Logic for Law Students: How to Think Like a Lawyer", at 16.

[110] Lovevinger, "An Introduction to Legal Logic", at 478.

3. **Argument a fortiori** - Relies on logical relations of inclusion within a class. This useful if you can draw sufficiently precise definitions of classes, but of limited use in a contextual domain such as law.[111]

Although legal reasoning does not come in the form of a syllogism, logic can still have a role to play in detecting and resolving inconsistencies, discovering similarities, and testing hypotheses.[112] Judges can be engaged in an interpretation of facts through the concepts and categories of the law, e.g. based on an existing framework for causality.[113]

Legal reasoning also involves induction - trying to encompass a range of diverse factual situations into a single normative assertion.[114] While science looks at inductive instances as the source of truth (which can be used as basis of its major deductive premises) law looks to the legislative power as basis of the truth of its major deductive premises, which then determines the scope of its inductive instances (in cases).[115]

Law can only continue to provide predictability if it is knowable. Simply enumerating actions and consequences (as was the case of primitive law) cannot scale to the growing complexity of society. Thus, the need to reduce law into a rational deductive system, so that we can infer conclusions based on a small base of axioms.[116]Although law can't be fully reduced into a deductive system like mathematics, legal scholarship should always strive towards that goal of systematizing the disconnected rules - finding commonalities, working toward consistency.[117]

[111] Ibid., at 478.
[112] Guest, "Logic in the Law", at 195-196.
[113] Samuel, "Is Legal Reasoning like Medical Reasoning?", at 341.
[114] E.g., a judge starts not with a comprehensive code, but with a constellation of prior decisions. Through induction, legal reasoning synthesizes these disparate particulars into a general legal formulation ibid., at 332.
[115] Gray, "The Ontology of Legal Possibilities and Legal Potentialities", at 9.
[116] Cohen, "The Place of Logic in the Law", at 624.
[117] Ibid., at 624.

3.2.9 A Balance Between Logic and Intuition

While law may not be a true science and hence not a site for reductive methods, it has a basic pattern for reasoning that is similar to logic: 1. From one case, we derive a proposition that serves as the rule of law; 2. The rule of law is applied to another similar case (and not to different cases). The human element comes in and can still affect the outcome, i.e. when the judge makes the determination of similarity or difference. There is no single rule that governs that determination.[118]

Logic is useful for analyzing, classifying legal materials but the content of such materials is based on social needs. Hence, a judge should neither be a logical automaton or a slave to psychological impulses but "should have logically and scientifically trained feelings".[119]

Despite the use of logical tools, there is a judgment calls involved in some modes of legal reasoning e.g.: 1. Bringing the facts of a case within a legal principle; 2. Reasoning by analogy. In both cases - one chooses to emphasize some things, omit others (facts, attributes). The judgment as to relevance implies some moral, ethical preference.[120]

3.3 Modern Approaches to Law and Logic

Computer scientists and philosophers have made many attempts to use logical tools to represent the intricacies of legal language and legal reasoning. This stream of work is based on the assumption that logic is a component of legal reasoning.[121]

[118] Lashbrooke, "Legal Reasoning and Artificial Intelligence", at 292-293.
[119] Lovevinger, "An Introduction to Legal Logic", at 474.
[120] Ibid., at 478.
[121] See generally Matthias Armgardt, Patrice Canivez, and Sandrine Chassagnard-Pinet, eds. *Past and Present Interactions in Legal Reasoning and Logic*. Vol. 7. Logic, Argumentation & Reasoning. Cham: Springer International Publishing, 2015. ISBN: 978-3-319-16020-7 978-3-319-16021-4. DOI: 10.1007/978-3-319-16021-4. URL: https://link.springer.com/10.1007/978-3-319-16021-4 (visited on 03/16/2024).

In legal theory (as well as AI research into the law domain), the logical aspects of legal reasoning is divided into two principal approaches:[122]

First, the formal approach - where legal decisions (e.g. the judge's justification) are arrived at through a mainly deductive process. Deductive reasoning draws conclusions from a set of general principles or premises that are given or established. This is related to formal symbolic logic.

Second, the dialectic (or argument theory) approach, which views legal justification as arising from an adversarial process, where parties use discretion to evaluate between reasonable alternatives.The approach borrows much from so-called "informal logic".[123]

The logical and dialectic approaches are seen as divergent, incompatible modes of legal reasoning, and for a long time have gone on separate tracks of development and application. The logical approach was seen as a tool for the legislative process, advancing the goal of representing laws as a set of consistent statements. Meanwhile, the dialectic approach was often applied to case-based problems that characterized litigation and judicial decision making - legal justifications derived from a process of presenting and evaluating pro and contra cases.[124] Nevertheless, advancements in both legal theory and technology may allow for the unification of the divergent approaches (of logic and dialectics). Within the case-based reasoning that defines the dialectic approach, there is acknowledgement that consistent logical rules can be formalized. Within the formal logic approach,on the other hand, researchers have developed models that take into consideration the incomplete and defeasible nature of legal argumentation. [125]

[122] Henry Prakken and Giovanni Sartor, eds. *Logical Models of Legal Argumentation*. Netherlands: Kluwer Academic Publishers, 1997. ISBN: 0-7923-4413-8. DOI: 10.1007/978-94-011-5668-4, at 1.

[123] Argumentation theory is based on informal logic, is not based on traditional syllogisms, where statements are evaluated based on logical validity (based on internal consistency, or whether or not the conclusion is structurally follows from premises), but is instead based on cogency (i.e. whether or not the premises are acceptable, relevant, and sufficient to ground the conclusion)

[124] *Ibid.*

[125] *Ibid.*

The foregoing analysis will cover debates covering the first approach. Much of the work in the field has emphasized the deductive approach, due to its seeming ubiquity in legal reasoning. The deductive approach is viewed as essential to legal interpretation and application: Lawyers will analyze the text, structure (and history) of a statute to determine meaning and intent. These will then serve, along with a background of other established rules, as premises for determining applicability to specific cases.[126]

[126] Jaap Hage. "A Theory of Reasoning and a Logic to Match". In: *Artificial Intelligence and Law* 4.3-4 (1996), pp. 199–273.

Chapter 4

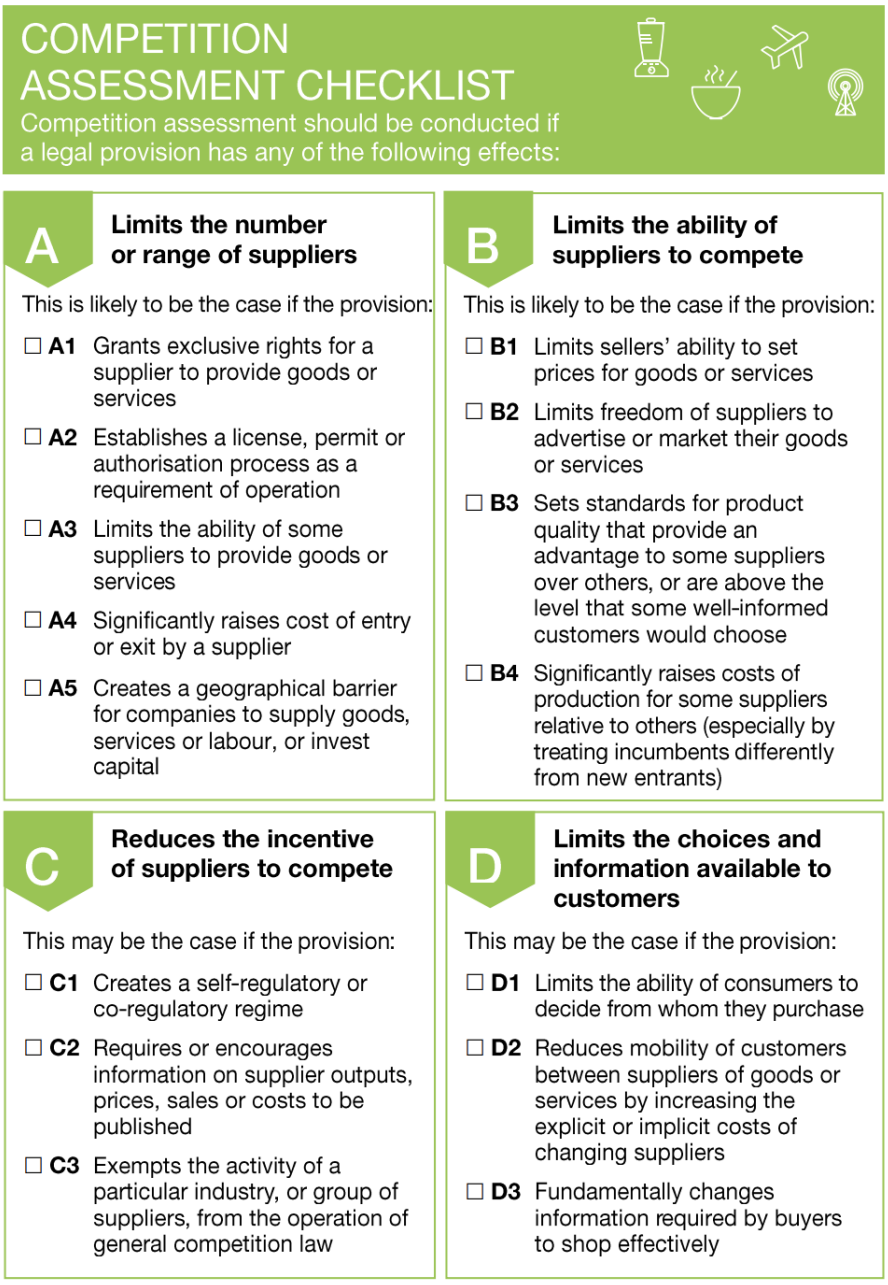
The Domain to Be Encoded - OECD Guidelines and Competition Rules

In order to provide a specific scope for the problem, and to provide formal requirements for programming, the study will need to define the domain of knowledge to be encoded. In cases where some concepts and rules are ambiguous, a description of the domain provides the opportunity to elaborate the choices for interpretation, as well as their bases.

This study is concerned with computational encoding of legal norms for competition analysis. As will be developed in subsequent chapters the effort will be focused on modeling the norms OECD Guidelines for threshold tests. The model will be divided into two main components: 1. An ontology which covers the definitions of entities and relationships in the rules; 2. A set of inference rules that capture the normative conditional requirements of these norms.

The OECD Guidelines provides fifteen norms divided into 4 subsections (see figure 4.1 below). Since, this study is concerned with providing proof-of-concept for encoding and analysis of competition norms, the scope is further narrowed to the norms under subsection A - norms that test against limitations to the number and range of suppliers.

Figure 4.1: The OECD Threshold Test Checklist



After providing a background of legal reasoning under competition law, this chapter will describe the legal and economic rationale behind the design and implementation choices for the encoding in subsequent chapters.

4.1 Normative and Legal Reasoning Under Competition Law

4.1.1 Underpinned by Economic Thinking

Competition law on the surface, is clothed with appurtenances of legal language (with reference to liability, burden of proof). It's internal logic, however, is guided by economics.Thus, competition law cases deal with facts not just as historical events but in terms of economic inferences or predictions (e.g. Does the merger create an incentive to raise prices?).

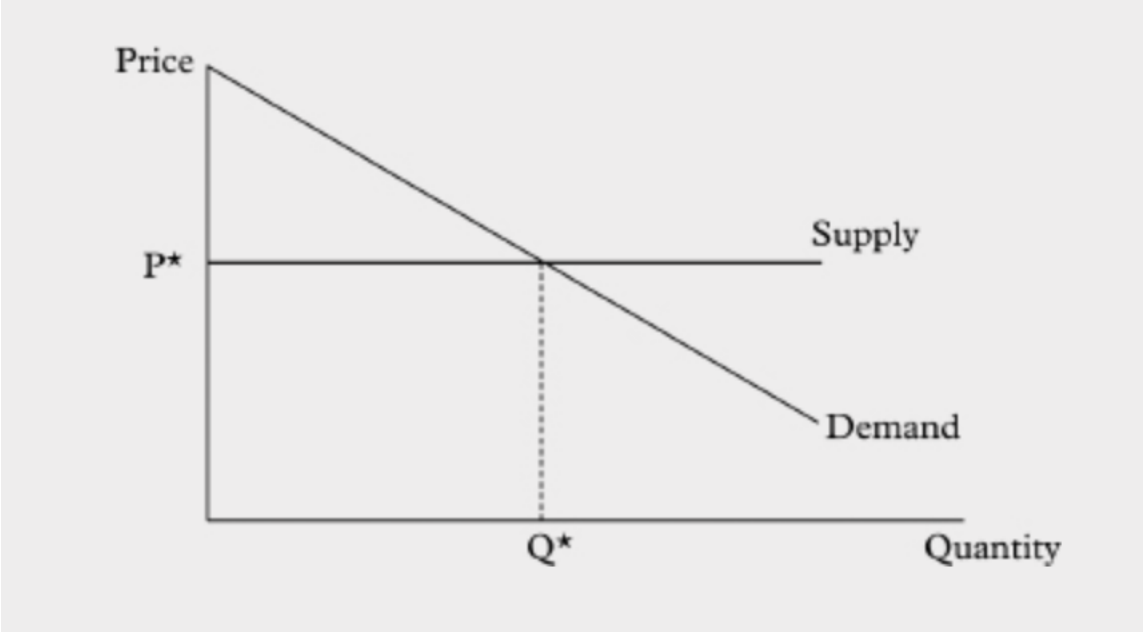
Competition law is underpinned by economic theory, which provides the normative basis for evaluating competitive behavior. It derives most of its rules from economic concepts and economic reasoning (particularly the model of competitive markets in modern neo-classical economics).[1] The basic premise is that more competition is better for consumers, as it leads to lower prices, greater innovation, and improved quality of goods and services. Competition law seeks to prevent anti-competitive behavior by firms, such as price-fixing, market allocation, and abuse of dominant position. The enforcement of competition law involves a complex interplay of legal and economic reasoning. Legal reasoning involves the interpretation and application of legal rules and principles, while economic reasoning involves the analysis of market structures, firm behavior, and consumer welfare.

Although there is much discussion in the economic literature about competition under various market conditions, the starting point of many policies is based on an idealized model of the market, e.g. a market for a homogenous product where there are many consumers and many suppliers, with no barriers to entry and exit and with good information, each participant, working towards their own self-interest will help deliver the optimal solution from society's perspective.[2] Figure 4.2 shows the interaction of demand for a product (which declines with price), with the supply of a product (which is constant at a price which reflects the underlying costs of the product)

[1] Helen Jenkins, Beatriz Yemail, and Peter Wills. "Economics at the Heart of Competition Policy". In: *Introduction to EU Competition Law*. New York: Informa Law from Routledge, 2013, at § 2.1.

[2] Ibid., § 2.1.

Figure 4.2: Model for Perfectly Competitive Market



In this case, the rivalry between consumers who want to purchase the goods, as well as the rivalry between suppliers who want to sell, results in output Q* being produced, sold at price P*. This is the price and output combination such that the quantity of output supplied by producers is equal to the quantity that consumers demand (the market equilibrium price)[3] The supply curve integrates conditions and assumptions about the market (e.g. homogenous product, no information assymetries/same technology level, and no barriers to entry or exit) The result is an unending supply of producers available to offer the goods at the price P*. Under this perfectly competitive market, any producer that increases its price would lose its business. Since: 1. Customers have no incentive but to chose anything but the lowest price product; 2. Other suppliers can step in, and expand their production to meet the demand.[4]

[3] Jenkins, Yemail, and Wills, "Economics at the Heart of Competition Policy", § 2.2.1.

[4] The above model would have firms competing only on price. In reality, firms can compete on a number of dimensions: quantity, product characteristics, research and development ibid., § 2.2.1, § 2.2.3.

4.1.2 EU Competition Law

In cases where the encoding will require additional semantics to provide additional details, the work will rely on the definitions and rules as elaborated in the EU competition law literature. This choice is motivated primarily by the following factors: 1. the correlation between EU competition law and the text of the Philippine Competition Act;[5] 2. the more extensive statutory text (in proportion to court decisions), especially in relation to the more sparse presentation of "Anti-trust" regime such as that found in the United States; 3. as will be argued later, recent developments in EU law point to a turn towards greater formalization (at least in some contexts) that will allow it to be more amenable to computational encoding.

The main objective of EU competition law is dealing with **market power**. Market power is defined as a firm's ability to make a profit despite raising prices above the level that would exist on a competitive market.[6] In the EU, competition law is also concerned with the functioning of the internal market. That is, preventing restrictions to the free movement of goods and services to and from other EU member states.[7] Major components of EU competition law include:

1. Art. 82, EC Treaty - Prohibits the abuse of positions of market dominance;
2. Art. 81, EC Treaty - Prohibits anti-competitive agreements;
3. EC Merger Resolution - Prohibits anti-competitive mergers, acquisitions, and joint ventures;[8]

4.1.3 EU Competition Law, Amenability to Formalization

There is tension in EU competition law enforcement approaches: form based (based on strict textual interpretation) versus "effects" based (i.e. based on economic effects). **Form based** refers to

[5] Many of the new concepts introduced in the Philippine Competition Act (such as "dominant position" and "relevant market") and covered behavior (e.g. abuse of dominant position, anti-competitive agreements), are derived from TFEU.

[6] Sean-Paul Brankin. "Introduction". In: *Introduction to EU Competition Law*. Ed. by Peter Wills. New York: Informa Law from Routledge, 2013, § 1.2.1.

[7] Piet Jan Slot and Martin Farley. *An Introduction to Competition Law*. 2nd Edition. Oxford and Portland, Oregon: Hart Publishing, 2017, at 2.

[8] Brankin, "Introduction", at § 1.4.

the use of formalistic categories in making legal determinations. For example, there are certain types of conduct (e.g. resale price maintenance, exclusive dealing by dominant firms - were presumed illegal per se based on their legal form).[9] The form based approach has been the historical mode of reasoning of EU competition law.[10] Syllogistic reasoning can manifest in the forms-based approach, e.g - IF a firm has a dominant position, AND IF it is engaged in conduct *x* (the legal category), THEN it is liable for abuse.

Later, this form based approach will come under criticism from the "Chicago School" of economics, which argued that many of the prohibited forms can actually be pro-competitive or efficiency enhancing.[11] According to Hildebrand, the period from 1990 to 2010 was characterized by a shift to a more economic, **effects-based** approach, where the EC adopted a methodology which requires enforcers to demonstrate that conduct actually harms consumers or excludes competitors that are just as efficient.[12] As can be gleaned from the 2008 Guidance on Enforcement Priorities - the EC shifted to an approach that leaned on economic effects, requiring enforcers to "take into account the dynamic nature of competition", and adopting counterfactual analyses - comparing the state of the market with the conduct under review to a hypothetical state of the market without the conduct.[13]

Miotto observes a recent turn back to formalism (or at least a structured formalism underpinned by economic theory) based on the 2024 Draft Guidelines on exclusionary abuses. There are indications of a shift away from a pure effects based approach and the employment of rebuttable assumptions.[14]

[9] See Justin Lindeboom. "Formalism in Competition Law". In: *Journal of Competition Law & Economics* 18.4 (Dec. 26, 2022), pp. 832–880. ISSN: 1744-6414, 1744-6422. DOI: 10.1093/joclec/nhac003. URL: https://academic.oup.com/jcle/article/18/4/832/6582875 (visited on 11/26/2025).

[10] See ibid.

[11] Marcus Glader. *Innovation Markets And Competition Analysis: EU Competition Law and US Antitrust Law*. New Horizons in Competition Law Economics. Massachusetts: Edward Elgar Publishing, 2006.

[12] Doris Hildebrand. *The Role of Economic Analysis in the EC Competition Rule*. 3rd. International Competition Law Series. The Hague: Kluwer Law International, 2009.

[13] European Commission. *Guidance on the Commission's Enforcement Priorities in Applying Article 82 of the EC Treaty to Abusive Exclusionary Conduct by Dominant Undertakings*. Feb. 24, 2009.

[14] Francesca Miotto, Maria Markou, and Janek Nowak. *From Effects to Presumptions? The EC Draft Guidelines on Exclusionary Abuse*. A&O Shearman. Aug. 12, 2024. URL: https://www.aoshearman.com/en/insights/from-effects-to-presumptions-the-ec-draft-guidelines-on-exclusionary-abuse (visited on 11/27/2025).

The shift back to formalism was necessitated by the failure of the effects-based by the failure of the effects-based approach in digital markets. While the economic effects can be easily observed in traditional markets (especially through the price mechanism), causality is not linear or easily traceable in digital markets. In digital markets, effect are emergent properties of complex systems. Proving a causal link between specific conduct and market outcomes in complex multi-sided markets is complicated, expensive and time consuming (often outpaced by developments in the market itself).[15] The European Court of Justice (ECJ) acknowledged the difficulty of the effects-based analysis in its *Google Shopping* judgment: While stating that causality is essential, the Commission cannot be required to establish a counter-factual where it is arbitrary or even impossible.[16]

4.2 The Rules for Encoding

As previously discussed,[17] the units of organization for the textual provisions (sections, paragraphs, clauses) may not correspond with the units of computational encoding and analysis - the if-then (normative conditional) inference rules. While it may be possible for a well-stated provision to directly state a single normative conditional statement, more likely, a provision can contain multiple inference rules. It may also be possible for a single inference rule to be distributed across multiple provisions.

Neither will the computationally encoded statements correspond with the identification and categorization schemes present in the text. This is especially the case when we refer to related texts and principles that further elaborate the rules. In the case of subsection A1 of the OECD Guidelines, for example - the norm, on its face, is simply a prohibition against exclusivity in suppliers. An initial, "naive" computational encoding can frame it as cardinality constraint, i.e. that the number of suppliers is $n = 1$. This may well be the "default" rule which corresponds to subsection A1.

[15] Andirani Kalintri. "EU Antitrust Law's Resilience: The Good, the Bad, and the Ugly". In: *Yearbook of European Law* 43 (2024), pp. 184–223. DOI: 10.1093/yel/yeae009, at 216.

[16] EU Court of Justice. *Google and Alphabet v Commission*. Sept. 10, 2024, at 37.

[17] See Chapter 1

However, as will be covered at length below, the OECD Guidelines suggest that it also reflects a broader norm against overweaning market power - which can in turn be correlated[18] with the EU competition law's prohibition against abuse of dominant position. Another example is subsection A2, which can be restated simply as a check on the presence of prior state authorization for the activities of a supplier. At the same time, it is linked to broader discussions on barriers to entry.

Thus, it may not always be possible to draw a one-to-one correspondence between the individual subsections and an encoded inference rule. A more accurate view the relationship between the text (and related context) of the OECD Guidelines and the encoded rules, is that the former is a pool of undifferentiated norms, which can be drawn upon to construct the inference rules. The OECD Guidelines can be seen as a checklist of factors that can be used to construct the inference rules, but the actual construction of the rules will require interpretation and elaboration based on the text and context of the guidelines.

4.2.1 Section A - Text and Context

A1 - Exclusive Rights A1 of the OECD Guidelines provides that a legal provision should be flagged for further competition assessment if it:

$$(A1) \text{ "Grants exclusive rights for a supplier to provide goods or services"} \tag{4.1}$$

The OECD Guidelines characterize exclusive rights as "the ultimate barrier to entry", leading to monopoly pricing.[19] It further elaborates that although it may not necessarily lead to monopolies, exclusive rights can represent entrenched market power and lead to suboptimal public welfare

[18] A further discussion of how encoded rules can be elaborated with more detailed, alternative interpretations, based on Robaldo (2020) is in Chapter 6 below.

[19] OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, at 10.

outcomes.[20] Market power (reckoned from the supplier side) refers to "the ability to profitably increase price, decrease quality, or decrease innovation relative to the levels that would prevail in a competitive market".[21]

The OECD's characterization of the rules under Section A suggests that the rules to be encoded should have a categorical constraint that tests for the presence of "barriers to entry", as well as a functional test that checks for abuses of market power.

This is supported by the PCC's CIA of the Mobile Number Portability (MNP) Act, which considers the potential abuse of dominance as an issue related to Section A of the CIA analysis (if not an outright barrier, then at least an anti-competitive aspect of market structure). In this case, while an exclusive right may not in itself be anti-competitive, the exclusive right can lead to market power dominance, which can be used to exclude other potential entrants. The likelihood of abuse may be considered as a factor for considering whether or not an exclusive right under A1 should be flagged.[22] Exploitative conduct, such as distorting the price mechanism, will not only affect consumer welfare but also shape the market to be less competitive.[23]

A2 - Licenses and permits A2 of the OECD Guidelines provides that a legal provision should be flagged for further competition assessment if it:

> ($A2$) "Establishes a license, permit, or authorization process as a requirement of operation" (4.2)

[20] The OECD Guidelines also suggest that the norm against exclusivity is not absolute, and that there may be circumstances where exclusive rights can be justified (e.g. to incentivize investment in infrastructure). ibid., at 10.

[21] Ibid., at 10, footnote 2.

[22] Olivo, Flores, and Bañez, "Competition Impact Assessment of the Mobile Number Portability Act (RA No. 11202)", at 11-12.

[23] Ibid., at 11.

The OECD Guidelines assert that the presence of licenses, permits, or authorization processes can create barriers to entry for potential competitors, which can lead to reduced competition and higher prices for consumers.[24]

The guidelines further elaborate that while such requirements may sometimes be necessary for certain industries and have well-founded consumer protection objectives (e.g. to ensure safety or quality standards). However, such barriers frequently have the effect of protecting incumbent producers from competition. Such processes should therefore be designed in a way that minimizes their impact on competition.[25] Licensing regimes can also be distorted from their legitimate purposes in a way that turns their application into a clear barrier to entry, creating long delay for issuance, opportunity for official corruption, or simply eliminating any realistic possibility of entry by worthy applicants for a license.[26]

As alternatives to licensing regimes, the OECD recommends considerations of the following regulations:[27]

1. Product quality standards that ensure consumer safety should not be set higher than necessary;
2. Restrictions on supplier size (e.g. only one storefront per professional), should not be set at levels that create substantial anti-competitive effects;
3. For requirements like compulsory insurance, performance bonds and the like - the extent of the damage should be taken into account;
4. Consumers can be protected from potential harm through more/better information;

A3 - Limitation of suppliers Under A3 of the Guidelines, a law that has the following effect should be flagged:

[24]OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, at 11.
[25]OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*, at 31.
[26]Ibid., at 31-32.
[27]OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, at 11-12.

(*A*3) "Limits the ability of some suppliers to provide goods or services" (4.3)

The prohibition applies to government acts that limit the ability of certain suppliers. The Guideline provides the following examples of these acts:

- Imposing a requirement real estate broker to provide a mandated set of services. This could limit or prohibit low-cost minimum-service brokers from participating;[28]
- Promoting regional suppliers, small suppliers, suppliers with special characteristics;[29]
- By limiting the ability of otherwise qualified suppliers;[30]
- Implementing a fertilizer subsidy program supply contract procurement required participants should have participated in the market for at least 5 years.[31]
- Proposal that port service providers need to own or lease land in the port area (this could lead to only large or established companies being able to secure scarce port area land);[32]

Regulations in this class can unduly restrict the number of suppliers, reducing competition between suppliers and lead to higher prices or less desireable contract terms.

The OECD recommends the following alternative policies, especially if the primary consideration is protecting regional economies or small businesses, alternatives:[33]

- Range of direct subsidies and tax benefits;

[28] Ibid., at 14.
[29] OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*, at 33.
[30] Ibid., at 33.
[31] Ibid., at 33.
[32] Ibid., at 33.
[33] OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, at 14.

- More favorable regulatory provisions for regional/small business;
- Publicity/educational campaigns.

A4 - Increases entry or exit costs Under A4 of the Guidelines, a law that:

$$(A4) \text{ “Significantly raises the costs of entry or exit”} \tag{4.4}$$

Violates the norm of the OECD Guidelines. Raising the cost of entry or exit - tends to discourage potential entrants, which will tend to reduce the number of market participants.[34]

Raising the cost of entry and exit discourages potential entrants and reduce the number of participants in the market over time.[35] It can also affect consumer welfare directly if the regulatory test for entry ends up substituting for market performance.[36]

Examples of regulations that may run afoul of A4 can include: 1. Disproportionately rigorous product testing requirements; 2. Unnecessarily high educational or technical qualifications; 3. For digital services in particular - requirements for physical presence in the jurisdiction as well as minimum scale.[37]

Alternative policy options offered by the OECD include: 1. Targeted exemptions, e.g. low volume manufacturers can be exempt from extensive testing regulations; 2. More information or product disclosure; 3. Maintaining an adequate degree of consumer protection for actual cases of harm.[38]

[34] OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*, at 12.
[35] Ibid., at 35.
[36] Ibid., at 35.
[37] Ibid., at 35.
[38] OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, at 12-13.

A5 - Geographic restrictions Finally, subsection A5 of the OECD tests is supposed to filter against:

$$(A5) \text{ “Restricts the geographic flow of goods, services, capital and labour”} \tag{4.5}$$

The prohibition is violated when the government prevents or limits flow of goods and services across jurisdictional boundaries.[39] Such course is usually pursued based on a policy of protecting a certain region, by limiting the area of competition.[40]

Beyond the reduction of the geographic area of competition of competition, the prohibited behavior is characterized by a reduction of number of suppliers, and/or allowing those remaining suppliers to exercise market power. This can ultimately result in limitation of consumer choice, and preventing an increase in productivity.[41]

In addition to a blanket, outright prohibition against cross-border flows, acts considered to flag under the test can include:

- A license regime (for selling tortillas in Tuxtla) which divides the city into four areas, and did not permit sellers to sell outside their designated area, thus creating four local monopolies;[42]
- A labeling requirement, if the effect is that consumers in an area cannot substitute a properly labeled product for a better or comparable product from another area that does not meet the labeling requirement.[43]

[39] OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*, at 36.
[40] OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, at 13.
[41] OECD, *Competition Assessment Toolkit - Volume 3 (Operations Manual)*, at 36.
[42] Ibid., at 37.
[43] Ibid., at 37.

4.2.2 Deriving Semantics from Barriers to Entry

Some parts of the encoding, such as those required for A2 and A5, are relatively simple and will be discussed first. A2 represents a specific legal requirement, embodied in a document or piece of data that allows a Supplier to offer goods or services within a particular market. A license requirement can also cover legal or regulatory requirements of proving public need prior to establishment or expansion. E.g. a hospital cannot buy new equipment to expand its services, or open a new clinic to serve a different area, unless it can overturn a prior determination that an area is already being adequately served. Thus, the modeling can involve determining whether authorization (usually represented through a document) is required prior to the supplier's offer of goods or services. Both the relevant objects (document, authorization) as well as the norm (if no document) is straightforward.

Likewise, A5 can be can be triggered through the definition of a geographic area, and a corresponding prohibition from offering goods and services imposed upon a supplier within or across that geographic area. Geographic areas can be defined not just in terms of physical coordinates (e.g. a geofence) but also legal determinations that jurisdiction applies to an area.

As will be elaborated in a subsequent chapter, we can encode A1 simply as a cardinality constraint - that is, a law should not entrench a single ($n = 1$) supplier or it can be viewed as a description of preferred market structure or behavior - that is, that there should be no player capable or likely to abuse its dominant position.

If we are to adopt the 2nd definition for A1, then the norm can be interpreted to check against a wide variety of behaviors - or even pre-existing conditions. Integrating "abuse of dominant position" may require us to model the many ways a supplier can be considered dominant. It can also require us to model the many paths available in the potential "strategy space" through which a supplier can abuse its dominant position - whether against consumers or other suppliers. Depending on the abusive behavior to be detected or controlled, the configuration of entities (consumers, suppliers,

contracts) and relationships can change. While we may be able to capture some of the more prevalent patterns of abuse, this may still fail to account for slight variations in the behavior, as well as new forms of exploitation.

Beyond taking into account specific scenarios and actions, it may be possible to evaluate the common signal across all of them, such as the imposition of disproportionate costs upon the supplier.

It is proposed that Section A is concerned about an evaluation of market structure - the number of suppliers participating, their relative strengths (i.e. in terms of market power), and the freedom of action (as a function of cost) they may have in terms of strategic behavior. While there are many possible factors that can contribute to these outcomes, both existing market power (as a component of "exclusivity" in A1) as well as inhibitions on the behavior of suppliers can be derived from the semantics of "barriers to entry", as a useful proxy variable. This is borne not just by the economic literature, but the competition assessment practice of the OECD as well as the PCC.

The same kind of simplicity cannot be said for any possible encoding of A3 and A4, which can be both more open ended. In the case of A3, the model should watch out against anything that can degrade a supplier's ability to provide goods and services. As stated in the previous section on scoping, the work is not concerned with an exhaustive encoding of the entire domain. In this chapter, we will demonstrate the encoding of the prototypical representations of the norm, some well-defined exceptions and other features of note.

Unlike the previous norms, A3 and A4 does not correlate with a finite set of requirements, but can encompass a wide spectrum of actions or affordances directed towards suppliers or to the market as a whole. It can extend to any possible act that can inhibit a Supplier's ability to offer its goods and services. Assuming that we limit it to government actions - this can still contemplate a broad area of possible actions.Inhibitions on a supplier's ability can come not just from monopolies (covered in A1) or license requirements, but also state aids and subsidies. price controls, as well as

discriminatory procurement (e.g. "buy local" mandates)

For both A3 and A4, the primary source of open-endedness is what constitutes the supplier's ability, which can be derived from so many factors. It may include its production or supply capacity (i.e. the capacity to increase output), its financial resources, and its technological capabilities. This can also determine what properly constitutes a cost for the supplier. A3 and A4 can be simplified and modeled jointly as a control against the law imposing significant costs (in the form of barriers to entry) - upon suppliers. This includes pre-entry cost in the case of A4, and post entry costs in the case of A3.

4.2.3 Barriers to Entry Under the EU Competition Law

The competition environment can be shaped by many factors, both from within the market itself (such as the technology level of the market players, their strategic behavior), as well as those that are exogenous to the market (such as government regulation). Each factor can be a potential source of . As stated in Chapter 1, however, the focus of the current work is on public conduct expressed through law. The norms that define barriers to entry is a suitable entry point for this analysis. Many barriers to entry can be represented in terms of (or arise from) public conduct - i.e., there is consensus that regulations and legal requirements are prototypical barriers to entry - A good example is a legal prerequisite to the participation in the market, e.g. a franchise or license requirements, covered by A2. We are thus primarily concerned with regulatory/government barriers since these are directly imposed, and traceable to legal text issued by a government authority. They are also exogenous to industry dynamics. This analysis can extend to those components of sunk costs (a recognized form of sunk costs) that are attributable to compliance with government regulations.[44]

Relationship with abuse of dominant position/market power While the literature on barriers to entry can provide a rich source of semantics, it will also be supplemented by the concepts and norms

[44]This will require our computational encoding to take into account the types of government regulations that impose a cost, and if these costs can constitute a barrier to entry.

related to the prohibition against abuse of dominance. There is convergence in the determinations made by barriers analysis and dominance analysis. For example - The presence of and extent of barriers determine the contestability of the market, and therefore how stable the position of a dominant firm. Furthermore, entry barriers can retard, diminish or entirely prevent the market's usual mechanisms for checking market power - the attraction and arrival of new competitors.[45]

Definitional debate on barriers to entry

Carlton asserts that the concept of "barriers to entry" has misled courts and regulatory agencies, especially in their application of the concept in antitrust cases of regulatory proceedings.[46]. The debate as to the meaning of barriers to entry is far from settled. The OECD only provides a general, if tautological definition of an entry barrier: "An impediment that makes it difficult for a firm to enter a market".[47] A barrier to entry is anything that hinders entry and has the effect of reducing or limiting competition.[48] Some scholars, on the other hand, have argued that an obstacle does not count as a barrier to entry unless it is something that the incumbent firms did not face when they entered".[49] Finally, there is a stream of thought which insists that definitional debates are irrelevant. What matters in actual cases are the effects: whether, when, and to what extent the likelihood of entry is affected.[50]

Bain's Definition of Barriers Bain defines a barrier to entry as anything that permits incumbents to earn above-normal profits without attracting entry. An advantage of an incumbent over potential entrants that can be measured by the extent to which incumbents can persistently hold their prices above competitive levels without attracting new firms to the market.[51] Bain's approach was based on finding exogenous factors of industry structure that influence how competition occurs and that

[45] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 17.
[46] Dennis W. Carlton. "Barriers to Entry". In: *Antitrust Law 2008*. Vol. 1. 2008, pp. 601–617, at 601.
[47] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 16.
[48] Ibid., at 16.
[49] Ibid., at 16.
[50] Ibid., at 16.
[51] See generally ibid.

prevent price from reaching the competitive level. Such Exogenous factors as barriers to entry include:[52]

1. Economies of scale - Include structural conditions, such as where the Minimum Efficient Scale (MES), is large relative to total demand.Since an entrant must either enter at a suboptimal scale with a cost disadvantage, or at an efficient scale with a depressing effect on prices;

2. Product differentiation - Describe conditions related to brand loyalty and preferences that favor incumbents (and constrains new entrants to incur significant marketing costs in order to divert demand). These can restrict entrance by allowing incumbents to charge higher prices than entrants and thus sell profitably when potential entrants could not;

3. Absolute cost advantages - These can come in the form of to superior production technology or exclusive access to key resources, or favorable financing terms, which allow incumbents to sell profitably at prices below the costs of potential entrants.

These barriers protect a firm from entry, allowing it to enjoy above normal rates of return.[53] Bain's work emerged from the Structure-Conduct-Performance (SCP) paradigm, which posited that market Structure (number of firms, barriers) dictates Conduct (pricing, innovation) which in turn determines Performance. For Bain, a barrier is an advantage of incumbent actors enjoyed relative to potential entrants. The advantage is reflected in the incumbent's ability to raise prices above competitive levels without attracting new firms to enter the industry. The definition is results oriented - it defines a barrier by its consequences - the ability to earn supra-normal profit.[54] Regulators adopting the Bainian framework would thus be wary of concentration of markets, with the assumption that features such as high capital requirements and advertising intensity are barriers to entry.

[52] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 239.
[53] Carlton, "Barriers to Entry", at 602.
[54] Carlton, "Barriers to Entry", See Generally; Citing Joe S Bain. *Barriers to New Competition*. Cambridge: Harvard University Press, 1965.

Criticisms of the Bainian Definition According to Carlton, Bain fails to articulate a model of causation that link his factors to elevated prices. It is based on a structure-conduct-performance view: Barriers determine number of firm, which leads to competitiveness of the industry, which in turn determines the firm's rate of return. This is too simplistic, since: 1. The number of firms is based on a decision to enter based on profitability; and 2. Profitability determine not just by exogenous factors (such as costs) but also by "vigor of competition". The vigor of competition can be viewed in game theory terms - the structure/type of competitive game being played. The impact of the barrier will depend on the type of game being played, and we cannot assume that this game is the same across all industries. High concentration may in fact be an indicator of a high degree of competition, and large size of forms is not necessarily associated with high prices. Therefore: The effect of structural factors may not affect equilibrium price in another industry.[55]

Some of Bain's factors, such as product differentiation, is not exogenous because it can be affected by investment - i.e., subject to the same economic forces that determine industry concentration.[56]

According to the OECD, Bain's definition is broad enough to include economies of scale and product differentiation, which are positively correlated with high profits. This has been controversial for being overbroad, being applied intuitively to anything that may have an anti-competitive effect. For example, a firm may be able to sustain supra-competitive prices just by having better products (e.g. absolute cost advantage).[57] At the same time, the Bainian definition may also be too narrow: It identifies no barriers to entry where incumbents earn no supra-competitive profits even though further entry is impossible. For example: There may be insufficient demand for even a solidly protected monopoly to cover its cost. It seems illogical to say that such a market has no entry barriers when we know that it does - yet this is the result of the Bain definition. This is due to using profit as the primary indicator for entry barriers.[58] Bain's definition also does not seem to recognize that in

[55] Carlton, "Barriers to Entry", at 602-603.
[56] Ibid., at 603.
[57] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 20-21.
[58] Ibid., at 21.

some circumstances it is more difficult for entrants to break into a market than it was for incumbent that was the first firm to enter. For instance: More precisely: when a market is already occupied by an incumbent, potential entrants might face an entrenched brand or brands, as well as demand that is insufficient to permit efficient operation. On the other hand, the first firms had no rival brands opposing it and could (even temporarily) enjoy the entire potential demand curve all to itself.[59]

The OECD traces the weakness of Bain's entry analysis not to inconsistent definition, but its failure to present a consistent theory showing that the factors that count as entry barriers lead to supra-competitive prices.[60] This in turn can be attributed to the theoretical and practical limitations of the underlying structure-conduct-performance paradigm, which may be too simplistic, since the number of firms in a market is determined by their ability to earn profits, which depends on many factors, not just structural ones.[61]

Stigler's Definition of Barriers Stigler defined an entry barrier as the "cost of producing... which must be borne by firms which seek to enter an industry but is not borne by firms already in the industry" - A cost advantage enjoyed by an incumbent firm, but not entrants.This cost advantage can be used by an incumbent to permanently elevate its price above its costs and therefore earn a supracompetitive return.[62]

Stigler has a stricter, cost-based definition - "A barrier to entry may be defined as a cost of producing (at some or every rate of output) which must be borne by a a firm which seeks to enter an industry that is not borne by firms already in the industry".[63] Under Stigler, scale economics cannot be considered entry barriers provided that entrants have equal access to technology - If the

[59] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 22.

[60] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 21; Citing Dennis W Carlton. "Why Barriers to Entry Are Barriers to Understanding". In: *American Economic Association* 94.2 (May 2004), pp. 466–470.

[61] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 21; Citing Carlton, "Why Barriers to Entry Are Barriers to Understanding".

[62] For example, if the incumbent firm has to spend $1 million annually to maintain brand reputation/customer loyalty, then as long as new entrant can do the same for an identical physical product, there is no reason to expect incumbent firms to earn supra competitive returns. Carlton, "Barriers to Entry", at 604.

[63] George J. Stigler. *The Organization of Industry*. Chicago: University of Chicago Press, 1968.

incumbent and the entrant have access to the same technology and cost function, the fact that the incumbent is currently larger and producing more cheaply is not a barrier, it is an outcome of the incumbent being there first. The entrant faces the same cost conditions the incumbent faced when it entered. The scale is not an asymmetry but a feature of the market. Furthermore, capital costs are not barriers, unless the incumbent never had to pay them.[64] For example, if advertising is correlated with high profits, Bain would consider it an entry barrier whereas Stigler would not, as long as advertising is available to everyone on the same terms - the Stigler definition is narrower.[65]

Criticisms of the Stiglerian Definition Carlton criticizes Stigler for not taking into account for dynamics or sunk costs,[66] and simply assuming a long-run steady state - i.e. that profits would naturally erode over time unless government regulation created asymmetries that protected incumbents (i.e. narrowed the scope of anti-trust enforcement, made more, bigger mergers possible). As a practical matter for lawyers and regulators, however, the promises of a long run equilibrium are meaningless. Although it became generally accepted accepted that sunk costs can be a source of entry barriers, these are excluded by Stigler's definition - since both incumbents and entrants may need to pay sunk costs before entering a market, they do not meet the requirements of a Stiglerian barrier.[67] One way sunk costs affect entry is by creating decisional asymmetry that is capable of deterring entry because incumbents have already paid them and entrants have not.[68] Such an asymmetry can arise because one firm is in a market before another firm and can therefore act to make binding commitments before others. For example, building a specialized, high capacity factory first signals the credible commitment to produce large outputs, and deter rivals from investing.[69] This in turn will affect what Carlton calls "dynamics" - the range of strategic behavior that firms can

[64] Provided that capital markets are efficient, if an entry opportunity is profitable, a firm will be able to raise funds. After all, the cost of capital is the same for both firms.

[65] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 21.

[66] Sunk costs - like fixed costs, does not vary with output. Unlike fixed costs, is not recoverable if the firm shuts down. An example, annual rent is a sunk cost if the firm cannot re-lease its office space pursuant to its lease if the firm ceases operation during the year (but not completely sunk cost if firm can recover all or part of the rent if it can sublet the space) Carlton, "Barriers to Entry", at 604.

[67] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 21.

[68] Ibid., at 22.

[69] Carlton, "Barriers to Entry", at 603.

adopt to gain advantage (since successful strategic behavior is based on asymmetry between firms). Based on game theory, sunk costs can inform us what credible commitments are most likely.[70]

While Schmalensee criticizes Stigler's definition as being too broad,[71] Demsetz maintains that both the Bainian and Stiglerian definitions are still too narrow: Even in the case of a prototypical regulatory barrier (such as a licensing requirement with limited number of licenses), Stigler's definition will not apply since the same cost of entry applied to both the incumbent and the new entrant. At the same time, Bain's definition may also not apply if market processes dissipate profits.[72]

Foregoing the Definitional Debates Both Bain and Stigler's definitions have animated the debate at both academic and policy levels. There are alternative definitions, some of which attempt to bridge the gap.[73] On the other hand,some commentaries suggest de-emphasizing the problem of definition altogether:"Rather than focusing on whether an 'entry barrier' exists according to some definition, analysts should explain how the industry will behave over the next several years."The emerging consensus from this line of thinking is that abstract, theoretical debate on the definition of barriers to entry is not helpful in actual investigations and policy decisions. Instead, the useful point of inquiry is whether and when entry is likely to defeat the anti-competitive effects of a decision or action. The OECD uses "barrier to entry" without adopting a particular definition. Instead, the term is used to denote anything that may be relevant to entry analysis as something that may prevent or delay entry, e.g. more generally as "conditions that may influence entry".[74]. The conditions that are of note for the OECD are:

[70] Carlton, "Barriers to Entry", at 606.

[71] This is under the assumption that variable costs are zero Organisation for Economic Co-operation and Development, "Barriers to Entry", at 21; Citung Richard Schmalensee. "Sunk Costs and Antitrust Barriers to Entry". In: *The American Economic Review* 94.2 (May 2004), pp. 471–475.

[72] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 21-22; Citing Harold Demsetz. "Barriers to Entry". In: *The American Economic Review* 72.1 (Mar. 1, 1982), pp. 47–57. JSTOR: 1808574. URL: https://www.jstor.org/stable/1808574, at 48.

[73] See R Preston Mcafee, Hugo M Mialon, and Michael A Williams. "What Is a Barrier to Entry?" In: *The American Economic Review* 94.2 (2004), pp. 461–465, McAfee, *et al.* makes a distinction between primary and ancillary barriers. Primary barriers are costs that constitute a barrier on their own (a patent, a government license). Ancillary barriers are costs that do not constitute a barrier individually but reinforce other barriers if present.

[74] Organisation for Economic Co-operation and Development, "Barriers to Entry"; Citing Carlton, "Why Barriers to Entry Are Barriers to Understanding", at 469.

1. **Sunk costs** - Sunk costs are investments that are fully committed to the market once they are made. Such costs cannot be recovered, even if the firm that invested them exits the business.[75] As discussed earlier, sunk costs can result in barriers to entry by creating decision and cost asymmetry between incumbents and potential new entrants: While the incumbent has already invested in the sunk costs necessary to enter market, the potential entrant has not.The potential entrant may also pay for the sunk costs, enter the market, and then immediately set a price level that is insufficient to recoup them (e.g. try to match the incumbent's strategy). For the potential entrant, however, the strategy can be irrational, since the potential entrant still has the choice not to lose the money it would spend on sunk costs. The incumbent, having already made the investment, no longer has the same choice. Thus, the incumbent can price at the level of its average non-sunk costs in order to deter entry, which shows that sunk costs may make entry unprofitable because of their effect on the post entry unit costs of incumbents. Therefore, unless the entrant is so much more efficient than the incumbent (so that it could price at or below the incumbent's non sunk cost level and still be profitable, all the time taking into account its own sunk costs). This may be too high a standard for most firms.[76] Furthermore, sunk costs interact with the background of uncertainty in any market to shape entry decisions: The higher the sunk costs are, AND the more uncertainty there is about risk condition, the less likely that a firm will enter. Consequently, if there are very low or no sunk costs, uncertainty will not be much of a deterrent. Because even in the case of failure, costs can still be recovered.[77]

[75] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 26.

[76] "When incumbents have already made irreversible investments that contribute toward manufacturing, distribution, sole of products, they will not necessarily take the cost of those investments into account when pricing in response to entry. These are not taken into account because: The money has already been spent and gone, and no course of action can ever recover it...Since money spent on sunk costs cannot be recovered, a firm that has already absorbed them can rationally ignore its sunk costs when making pricing decisions.Thus: An entrant facing an incumbent who has invested in substantial fixed cost is facing a rival that might respond to entry by cutting its prices below the point where it is covering its sunk cost. The post-entry price competition from incumbents who have substantial can thus be "fierce", i.e. such incumbents may be able and willing to deter entrants that would be less efficient, as well as those just as, or even those more efficient than incumbents." Organisation for Economic Co-operation and Development, "Barriers to Entry", at 27. Citing Stephen Martin. "Sunk Cost and Entry". In: *Review of Industrial Organization* 20 (2002), pp. 291–304.

[77] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 28.

2. **Structural barriers** - These arise from basic industry conditions (costs, demand, and technology). These factors are usually beyond the incumbent's direct control, or are byproducts of their efforts to compete in general, rather than specific strategies deployed to deter entry.[78] These can include cost advantages,[79] high capital costs,[80] network effects,[81] legal restrictions, and economies of scale.[82] The concept of structural barriers also includes legal and regulatory barriers, which will be a focus of the encoding and will be discussed at length below.

3. **Strategic behavior** - While sunk costs and structural barriers are based on background factors, the OECD notes that we also need to account for the actions of the incumbents. Barriers to entry based on strategic behavior of incumbents are entry conditions that were intentionally created or or enhanced by the incumbent, often (but not automatically or solely) for the purpose of deterring entry.[83] Such practices include predatory pricing,[84] loyalty rebates/bundling,[85] exclusive dealing arrangements.[86] Since the focus of this work is evaluating public conduct expressed through laws, the encoding will largely not account for the strategic behavior of

[78] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 28.

[79] Cost advantages exists when an incumbent's cost curve is below an entrant's at every level of output. As long as the incumbent prices below the entrant's expected cost, the incumbent can earn supra competitive profits, yet entry will remain unprofitable. ibid., at 28.

[80] High capital costs has two components:1. Absolute magnitude of the total costs necessary to enter the market; and 2. The relative cost of borrowing the money to fund entry. Some scholars contend that high capital costs cannot be considered barriers because well-functioning capital markets will fund any firm with a reasonable entry plan at fair interest rate.However, the higher the capital costs required to enter a market, and the greater the proportion of these costs are to be sunk, the riskier a potential entrant will consider entry to be, since it stands to lose more money if the business fails. Likewise,the impact that high capital costs are likely to have on a potential entrant's assessment of risk depends on its particular situation: A new entrepreneur with no other business and no experience is more likely to be daunted by high capital costs than an established conglomerate. ibid., at 30-31.

[81] Network effects arise when the benefit a consumer receives from a product increases not only with the amount of that he or she consumes, but with the number of other people who are availing of it as well. ibid., at 31.

[82] Economies of scale are in effect when average costs decrease as units of output increase over a certain range of production. Within that range of production, even if incumbents and entrants have the same cost curves, the incumbent may be able to operate at higher output levels where costs are lower than they are at a potential entrant's expected level of output, given the likely differences in demand. ibid., at 29.

[83] Ibid., at 35.

[84] An incumbent willing and able to price below cost for an extended period of time, may be able to deter entry. A related strategy is limit pricing, where incumbents do not pricing below cost but purposefully fail to maximize short-run profit in the short term. By sacrificing some profit, the incumbent can deter entrants and therefore earn supra-competitive profit. ibid., at 35.

[85] Incumbents can establish a barrier by granting discounts to loyal customers, or by conditioning rebates on the purchase of another products. ibid., at 36.

[86] I.e., the more demand that an incumbent has to the exclusive right to supply, the more difficult the entry. ibid., at 38.

market participants, even if such behavior create or magnify barriers to entry.

Laws as barriers to entry Proponents of the Chicago School view regulatory or legal restrictions imposed by governments as the most substantial of entry barriers. At the same time, we cannot dismiss regulations automatically since these legal provisions may well serve other important policy goals, including competition.[87] However, some government-imposed restrictions such as tariffs and import quotes have the express purpose of raising entry barriers and limiting competition.[88]

Sunk costs and regulatory barriers are viewed as the two primary sources of entry barriers, as they bear heavily on other factors that can impede entry.[89] The OECD considers regulations primarily as a structural barrier to entry, and this distinct treatment should be taken into account by the encoding. However, to the extent that the law can impose sunk costs, such features of the law should be taken into consideration by the computational model. Likewise, the encoding should also take into account the possibility that law may enhance other structural barriers. For example, "...permanent cost advantages could arise from a requirement that new entrants buy certain environmental protection equipment that incumbents are not required to buy".[90]

Finally, although it is beyond the scope of this work to model the entire panoply of strategic behavior of individual market participants, it will take into account the possibility that laws can establish or magnify market power to the extent that it becomes a barrier to entry. The detection and measurement of market power becomes important for two reasons: First, it serves as a more realistic representation for exclusivity under A1; Second, it provides an additional indicator for whether or not the law is likely to give rise to a barrier, enough to activate the thresholds for A2 and A4.

[87] Some of these barriers may even benefit competition and consumer welfare. E.g. A patent may prevent rivals from entering a market by imitating the incumbent's product, but the product might not have existed in the first place without the patent protection. ibid., at 33.

[88] Ibid., at 34.

[89] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 26. Citing Thomas Ross. "Sunk Costs and the Entry Decision". In: *Journal of Industry, Competition and Trade* 4.2 (June 2004), pp. 79–93. ISSN: 1566-1679, 1573-7012. DOI: 10.1023/B:JICT.0000037355.82246.e0. URL: https://link.springer.com/10.1023/B:JICT.0000037355.82246.e0 (visited on 04/02/2026).

[90] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 28.

Barriers to Entry in Digital Markets

The digital economy has necessitated a fundamental re-evaluation of traditional barrier theories.[91] While physical capital requirements for digital startups are often low, new types of intangible barriers have emerged that are potentially more durable and harder to measure. For example, established platforms utilize user data to train algorithms (e.g. for search ranking ad targeting).[92] Better algorithms attract more users, who provide more data, which further improves the algorithms. This creates a "virtuous cycle" for the entrant. Even with superior code, an entrant cannot replicate the historical data training set required to offer a competitive product. Digital markets are often characterized by strong network effects, where the utility of a service is proportional to the size of its user base. Network effects can entrench incumbents and impose costs on new entrants.

A dominant platform can often act as a dual role player i.e. it both operates the marketplace and sells products on it (e.g. Amazon Basics on Amazon). In its capacity as the market operator, it may engage in self-preferencing - the practice of favoring one's own services in rankings or interface design is viewed as a behavioral barrier. It denies rivals visibility needed to achieve a minimum viable scale. If an entrant cannot access the platforms users on fair terms, the platforms itself acts as a barrier to the downstream market.[93]

[91] See generally Daniel Mandrescu. "Applying (EU) Competition Law to Online Platforms: Reflections on the Definition of the Relevant Market(s)". In: *World Competition: Law and Economics Review* 41.3 (2018), pp. 453–483, Mandrescu notes the rise of so-called platform businesses, their disruption of traditional understanding of markets and the applicability of competition law.

[92] Robert D. Anderson et al. "Competition Policy and the Global Economy: Current Developments and Issues for Reflection". In: *George Washington Law Review* 88 (Nov. 2020), pp. 1421–1476, at 1436 - 37.

[93] See Moira Weigel. *Amazon's Trickle-Down Monopoly: Third-Party Sellers and the Transformation of Small Business*. New York: Data & Society Research Institute, 2023. URL: https://datasociety.net/wp-content/uploads/2023/01/Weigel_Trickle-Down-Monopoly_01252023.pdf (visited on 11/16/2023), Weigel provides an account of competition impact (from digital platforms) that diverges from the consumer welfare theory. Although prices for the ultimate consumer of retail goods has not been adversely impacted during Amazon's tremendous growth, Amazon's size and competitive behavior have impacted the market in more subtle, often structural ways. Amazon has continuously introduced new service or regulations, or has directly competed with 3rd party sellers - and these behaviors have placed significant constraints on many smaller 3rd party seller, even eliminating their businesses. Amazon has transformed the retail market, enrolling 3rd party sellers to expand its own influence, and at the same time infecting these with their own monopolistic behavior - mistreatment of contractors and employees, stockpiling to control supply, selling at a loss in order to corner niche markets, and making their businesses less transparent.

There is a shift in competition norms, represented by the EU's Digital Markets Act,[94] where instead of proving barriers on a case-by-case basis, recognizes that they can interact and converge in the form of firms that have a gatekeeper status. This status is presumed based on the firm's size as well as its position in the market.

4.2.4 Abuse of Dominance Under the EU Competition Law

Definition based on market power Article 102 of the TFEU[95] prohibits the abuse of dominant power either through unilateral conduct of a single dominant firm or independent action of several oligopolists. It is the abuse of a dominant position that is prohibited, rather than the dominant position itself.[96] In interpreting Art. 102 of the TFEU,[97] competition authorities may (with caution) draw on principles from other areas of competition law, such as Art 101, merger decisions, which may share common elements such as dominance in neighbouring markets, portfolio power, and bundling.[98] Under EU competition law, a dominant position is "a position of economic strength enjoyed by an undertaking which enables it to hinder the maintenance of effective competition on the relevant market by allowing it to behave to an appreciable extent independently of its competitors and customers and ultimately of consumers".[99] It is characterized by sufficient economic strength to act independently of its customers and competitors to a significant extent. Thus, the concept of dominance approximates significant market power. Dominance results in the maintenance of prices above competitive levels, and incentivizes the anti-competitive behavior of private parties, such as pricing behavior intended to exclude potential competitors from the market or constrain the growth of existing competitors (e.g. predatory pricing).[100]

[94] European Union. *Digital Markets Act*. Dec. 10, 2022.

[95] Previously Article 82 of the EC Treaty

[96] Martin Baker, Taylor Wessing, and Pat Treacy. "Article 82: Principles and Application". In: *Introduction to EU Competition Law*. Ed. by Peter Wills. New York: Informa Law from Routledge, 2013, at § 5.1.

[97] Previously Article 81 of the EC Treaty

[98] Martin Baker, Taylor Wessing, and Pat Treacy, "Article 82: Principles and Application", at § 5.1.

[99] Ibid., at § 5.3, citing Case C-322/81, *Michelin NV v. Commission* (1985) 1 CMLR 380.

[100] See Jenkins, Yemail, and Wills, "Economics at the Heart of Competition Policy", at § 2.2.2.

Factors for determining dominance

Determining whether or not a firm enjoys a dominant position requires determination of the following factors:[101]

1. The "relevant market"

2. The relevant firm's "position of economic strength" in the relevant market

Assessment of dominance is based on finding characteristics (or lack of it) of a competitive market as outlined by underlying economic theory. This means looking for: 1. Evidence that prices are above cost; 2. Indicators of market structure (market shares, entry, exit, and barriers to entry and exit, consumer switching); 3. Unilateral pricing power of the firm in question.[102]

Relevant market Market definition is crucial to the determination of whether a dominant position exists, since the narrower the market, the greater the likelihood of an undertaking holding a position of market power.[103] The relevant market can be defined as "The market in which the dominant undertaking operates for the purposes of competition law".[104] A relevant market can include:

1. **The product market** - Comprises all those products and/or services which are regarded as interchangeable or substitutable by the consumer, by reason of the product's characteristics, their prices, and their intended use.[105]

[101] Brankin, "Introduction", at § 1.5.1.

[102] Jenkins, Yemail, and Wills, "Economics at the Heart of Competition Policy", § 2.2.2.

[103] Martin Baker, Taylor Wessing, and Pat Treacy, "Article 82: Principles and Application", § 5.2.

[104] Market definition - done through identifying the "scope of potential competitive constraints on the undertaking alleged to be dominant". This essentially involves determining who the firms's competitors are. Brankin, "Introduction", at § 1.5.1.1.

[105] The relevant product market can also include products that are not direct substitutes (supply-side or demand side substitution).Citing *Market Definition Notice*, 1997 OJ C 372 p. 5 para 7. Brankin, "Introduction", at § 1.5.1.1.1. The product market is the market for all product and/or services in question that are regarded as interchangeable, or substitutable, by the consumer by reason of their use, price, and characteristics. Product markets tend to be defined narrowly. See *BBI/Boosey & Hawkes* (1988) 4 CMLR 67, relevant product was defined as "brass band market for British-style brass bands" instead of market for brass bands as a whole. Citing Martin Baker, Taylor Wessing, and Pat Treacy, "Article 82: Principles and Application", § 5.2.1.

2. **The geographic market** - This is the area in which the undertakings concerned are involved in the supply and demand of the products or services, in which the conditions of competition are sufficiently homogenous and in which can be distinguished from neighboring areas because the conditions of competition are appreciably different in those areas.[106]

Analytical tools for determining relevant market A prevalent analytical tool for determining the relevant market is the Small but Significant Non-Transitory Increase in Price (SSNIP) test. This is an iterative process in which all products and geographic regions that provide a significant constraint on a small set of products and regions are identified:[107]

1. Step 1 - Identify a narrow set of products in a geographic area;

2. Step 2 - Determine whether a hypothetical profit-maximizing monopolist with control over all the products in the area identified in Step 1 would be able to raise prices profitably by 5-10 per cent for a period of time of around 1 year;

3. Step 3 - If the answer to Step 2 is NO, iterate the test from Step 1 with additional products and regions - to identify the set of products worth monopolizing.

In determining a firm's ability to raise prices, consideration should be given to demand-side substitution[108] as well as supply-side substitution.[109]

The SSNIP test requires modeling a general demand system as well as making econometric estimates of all own- and cross-price elasticities of demand (using empirical price and sales data),

[106] Brankin, "Introduction", at § 1.5.1.1.2. Geographic market is the area in which the undertakings in the supply and demand of products or services in which the conditions of competition are sufficiently homogenous, and which can be distinguished from neighboring areas because the conditions of competition are appreciably different in those areas. Martin Baker, Taylor Wessing, and Pat Treacy, "Article 82: Principles and Application", at § 5.2.2.

[107] Jenkins, Yemail, and Wills, "Economics at the Heart of Competition Policy", at § 2.3.1.

[108] A price increase will not be profitable if a large enough portion of the hypothetical monopolists customers can easily switch to cheaper products or products with similar characteristics, or that can be used for the same purpose. ibid., at § 2.3.2.

[109] A price increase will not be profitable if existing companies can supply similar products, or the same products in nearby geographic sectors. ibid., at § 2.3.2.

making it difficult to apply.[110] Another limitation of the test is that it may fail to properly define a market where market power already exists. For example, if the market is already monopolistic, customers will switch to other products that are not necessarily substitutes, in the event of a price increase. In such a market, customers will switch to other products that are not necessarily substitutes, in the event of a price increase.[111] The test can be substituted or supplemented with price correlations over time, or surveys asking consumers if they are likely to switch given a price increase.[112]

Determining dominant position After determining the relevant market, the next step is to assess the firm's position in that market - that is, the degree to which the firm can act independently of (or is in fact constrained by) its competitors.[113] An evaluator will need to decide if the undertaking is dominant in those markets. While dominant undertakings are not necessarily monopolists, monopolists will generally have a dominant position.[114] Dominance is assessed on a case-by-case basis depending on a number of factors, not one of which is conclusive.[115] Factors for assessing dominance may include: high, stable market shares (especially in relation to competitors); access to raw materials and capital; technical knowledge and expertise;barriers to entry; existence of any abusive behavior.[116]

Market share is a significant factor in analyzing dominance. Although there are other factors, as a general rule, the larger the firm's market share, the more likely that it is holding a dominant position.[117] EU Competition Law has developed a system of market share thresholds for analyzing the presence of dominant position:[118]

[110] Jenkins, Yemail, and Wills, "Economics at the Heart of Competition Policy", at § 2.3.2.
[111] Ibid., at § 2.3.2.
[112] Ibid., at § 2.3.2.
[113] Brankin, "Introduction", at § 1.5.1.2.
[114] Martin Baker, Taylor Wessing, and Pat Treacy, "Article 82: Principles and Application", at § 5.3.1.
[115] Ibid., at § 5.3.1. Citing *Commission Guidelines on market analysis and the calculation of significant market power*, 2002 OJ C 165, p. 6.
[116] Ibid., at § 5.3.1. Citing Case 27/76, *United Brands v. Commission* (1978), 1 CMLR 429, Case T-203/01, *Michelin II v. Commission* (2004) 4 CMLR 923.
[117] Brankin, "Introduction", at § 1.5.1.2.
[118] Ibid., at § 1.5.1.2. Citing See *Virgin/British Airways*, (2000) OJ L 30, and Case C-62186, *Akzo v. Commission* (1991) ECR I 3359.

1. **10%** - The market share is too small to be considered as dominant;[119]

2. **20%** - There is a possibility of dominance, if considered with other factors;[120]

3. **25-40%** - A firm with more than 25% less than 40% market share is unlikely to be found to hold a dominant position on its own, unless there is a fragmented market and other significant factors;[121]

4. **40% or more** - Evidence of dominance can be considered with other factors,[122]

5. **More than 50 %** - A firm with stable market share above 50 per cent will be presumed dominant provided that an examination of other relevant factors does not indicate that it is not;[123]

6. **75% or more** - If a firm has in-excess of 75% market share, the firm may be found dominant on that basis alone;[124]

7. **85-90%** - A firm with a market share of 85% will be considered to be conclusively dominant unless there are countervailing factors.[125]

2202 These thresholds are not absolute rules but provide guidelines for courts and regulators. Other
2203 factors could include the relevant market; the stability of the market share over time.[126] Relative
2204 size of the market shares of relevant firm's nearest rivals is also a relevant consideration, since

[119]Citing Case 75/84, *SABA II Metro SB v. Commission, (No 2)* (1987), 1 CMLR 118. Martin Baker, Taylor Wessing, and Pat Treacy, "Article 82: Principles and Application", at § 5.3.2.

[120]Citing *Metaleurop*, 1989 OJ C 100, p. 2; (1989), 4 CMLR 745. ibid., at § 5.3.2.

[121]Citing *Grundig*, 1985 OJ L 233, p 1; (1988) CMLR 865. ibid., at § 5.3.2.

[122]Citing Case C-85/76, *Hoffmann-La Roche* (1979), 3 CMLR 211. ibid.

[123]Citing Case C-62/86, *Akzo v. Commission* (1991), ECR I 3359. Note that other factors were also regarded as relevant, See Case T-395/94, *Atlantic Container Line v. Commission* (2002), ECR II 595 and Case 322/81, *Michelin v. Commission* (1985), 1 CMLR 380. ibid., at § 5.3.2.

[124]Citing Case C-85/76, *Hoffmann-La Roche* (1979), 3 CMLR 211. ibid., at § 5.3.2.

[125]Citing Case C-85/76, *Hoffmann-La Roche* (1979) 3 CMLR 211. But see merger cases such as Case M.068, *Tetra Pak v. Alfa-Laval*, 1991 OJ L 290, p. 35, where even a market share of 90 per cent did not confer dominance. In such cases countervailing buying power is often a relevant factor: see Case M.1225, Enso/Stora (1998) Reports on Compensation Cases 54 and Case M.1684, *Carrefour/Promodes*. ibid., at § 5.3.2.

[126]According to the ECJ, high market share for 3 years is relevant. See *Akzo v. Hoffman-La Roche*, Case 85/76, (1979) ECR 461.

weaker, smaller competitors means they are less capable of exerting any real competitive pressure on the relevant undertaking.[127] The undertaking should have the ability to maintain market share, through their market power, in the long term. Fluctuating market share tend to suggest an absence of dominance. The period of time required based on circumstances.[128] Consequently, high market shares in a new market are not sufficient indication of dominance, since the share could still be fluctuating.[129] Where an undertaking has dominance on a specific market, it may also be deemed to have dominance on a closely related market, if the markets are linked in such a way that market power in one can be leveraged for the other. If such undertaking is able to exercise leveraged market power over a market other than the one it currently dominates, this may be a violation of Article 102, if it abuses its position on the other market.[130]

Establishing abuse Having a dominant position does not necessarily violate Article 102. In order to qualify as a violation (and consequently, to constitute a barrier under the computational model), a firm not only has to possess, but also abuse said dominant position.[131] Abuse should be distinguished from ”normal competition”, that is seeking to gain customers by satisfying their demand for cheaper, better products.[132] EU courts have not provided rigorous legal definition to abuse of dominant position, instead determining on a case-by-case basis whether particular conduct is abuse.[133] There are nevertheless general categories of abusive behavior that the courts recognize:

1. **Behavior establishing or increasing market power** - When the firm increases its existing level of dominance, or uses its existing dominance in one market to establish a new position of dominance in another market.[134] An example of such behavior is predatory pricing -

[127] Martin Baker, Taylor Wessing, and Pat Treacy, “Article 82: Principles and Application”, at § 5.3.3. Citing Case 85/76, *Hoffman-La Roche v. Commission* (1979) ECR 461.

[128] Ibid., at § 5.3.4. Citing Case T-65/98, *Van den Bergh v. Commission* (2004), 4 CMLR 1.

[129] Ibid., at § 5.3.4. Citing Case IV/M.469, *MSG Media Services*, (1994) OJ L. 364.

[130] Ibid., at § 5.3.5. Citing Case C-333/94 P *Tetra Pak International SA v. Commission (Tetra Pak II)* (1994), ECR II 755. The Court ruled that Tetra Pak not only a quasi monopoly on the (dominated market) but also held a strong position on the associated market. Customers on one market were potential customers on the other.

[131] Brankin, “Introduction”, § 1.5.2.

[132] Citing *Hoffman-La Roche v. Commission*, *supra*, par. 91 ibid., at § 1.5.2.

[133] Ibid., at § 1.5.2.

[134] Ibid., § 1.5.2.

Once competitors are eliminated by the artificially low price, the dominant firm is no longer constrained from raising prices.[135]

2. **Behavior entrenching market power** - When a dominant firm does something to protect its existing position of dominance from competitive challenges, e.g. by placing barriers to entry.[136] An example of this behavior would be offering a loyalty bonus, i.e. discounts conditioned on customers buying all or most of their required products from the dominant firm. This makes it harder for new entrants to acquire customers or enter the market.[137]

3. **Behavior exploiting market dominance** - This involves the firm using the dominant position it gained to either exploit its customers through excessive prices, or through the exclusion of its rivals.[138] To qualify as excessive, prices must "charging a price which is excessive because it has no reasonable relation to the economic value of the product."[139] Exclusionary conduct can take the form of:

    - **Predatory pricing** - Where where a dominant firm deliberately incurs short term losses in order to eliminate competitors. This usually requires that the price be below cost and entry barriers are already high.[140]

    - **Margin squeeze** - Where a vertically integrated firm dominates the market for an input to a downstream product that is important not only for itself but for other rival firms in the downstream market. The dominant vertically integrated firm lowers it downstream price or raises the price of the input. The effect either way: Margins earned by the rival is lowered to the point they cannot compete profitably. This will force the rival off the market, which will enhance the integrated firm's dominant position.[141]

[135] Ibid., at § 1.5.2.1.
[136] Ibid., at § 1.5.2.
[137] Ibid., at § 1.5.2.2.
[138] Jenkins, Yemail, and Wills, "Economics at the Heart of Competition Policy", at § 2.3.3.
[139] Citing Case 27, *United Brands v. Commission* (1978), ECR 207, 1 CMLR 429 ibid., at § 2.3.3.1.
[140] Ibid., at § 2.3.3.2.
[141] Ibid., at § 2.3.3.2.

- **Bundling** - Occurs when new entrants a re forced to offer additional products. If products are complementary, new entrants are constrained. Distorts competition when existing rivals and new entrants are prevented from offering competing bundles.[142]

4.3 Considerations for Modeling Barriers

4.3.1 Characterization of Barriers Under PCC Practice

The PCC's competition impact assessment work (its methodologies and findings) are contained in periodic Market Studies and Issues papers, available through its website.[143] This body of work is considered the baseline for reckoning improvements from a computational approach, as well as a source of interpretive rules to be encoded. The body of work was analyzed for:

1. How they define sectors to be studied;
2. How they select relevant laws for analysis;
3. Their units of analysis for legal reasoning;
4. Their interpretation of the competition effects of the relevant law.

Through the last item above, these works also communicate the agency's sense of what constitutes barriers to entry for specific markets. Such notions can be a source of both semantics and norms for what constitutes barriers to entry especially for specific markets:

- **Nationality requirements** Limitations on foreign ownership for cargo services actors such as: freight forwarders, storage and warehousing services, such as citizenship requirements for ownership and executive control are a barrier to entry.[144] Another aspect of this barrier is

[142] Jenkins, Yemail, and Wills, "Economics at the Heart of Competition Policy".
[143] https://www.phcc.gov.ph/resource-category/issues-papers
[144] Llanto and Francis Mark A. Quimba, "Competition Policy Issues in Cargo Services", at 21.

the discrepancy of treatment between domestic and international carriers (e.g. in terms of tax);[145];

- **Complexity of license process** Even assuming that the licensing criteria is open enough to not be a barrier in itself, the approval process - e.g. the multiplicity of steps and licensing authorities, can constitute a barrier to entry;[146]

- **Structural barriers** Structural barriers in the cargo services sector could include: 1. Substantial capital costs; 2. Availability of specialized skills (particularly for port/customs operations and administrative requirements);3. Network of contacts to ensure continuing demand for services;4. Access to technology;[147]

- **Import prohibition**The requirement that **all** fuel products for motors and engines contain locally sourced biofuels amounts to a prohibition against imported biofuels that creates a barrier to entry for the local fuel market;[148]

- **Access to data** Data interoperability promotes more competitive markets by expanding the array of services on platforms, enriching the user experience. It can reduce the cost of switching between platforms.Therefore, the lack of (or practices against) interoperability may have to be flagged as a barrier to entry;[149] Competition concerns could also arise from firms getting more access to cross-border data, not only gaining competitive advantage, but also limiting the ability of their competitors to compete. Vast pools of data can act as "protective moats" that can entrench incumbents.[150] If a new entrant cannot collect the data or access the same volume or variety of data. For example: Established players operating at a global scale, can benefit from greater value from integrated user data collected across borders. Their

[145] Ibid., at 26.
[146] Ibid., at 21-22.
[147] Ibid., at 13.
[148] U et al., "Market Study on the Refined Petroleum Industry".
[149] Abrenica, Ramos, and Cleofas III, "Competition Policy and Data Privacy in Social Media and Search Platforms".
[150] Olivo, Flores, and Bañez, "Competition Impact Assessment of the Mobile Number Portability Act (RA No. 11202)", at 12.

competitors, on the other hand, will have limited access to such data and will not be able to enhance their products at the same level;[151]

- **Network effects** In digital platform markets (such as search and social media), network effects interact with market share and economies of scale/scope in order to entrench incumbents.[152] In the case of direct network effects, the value of a platform for a user increases as more of their contacts join the platform - thus, the larger user base, the greater its has more value, and more likely to attract more users.[153] The platform's growth in "winner take most" manner lends it economies of scale, i.e. once a digital platform has grown to a large scale, if becomes more difficult for entrants to compete, due to high fixed costs associated with initial development, coupled with low marginal cost for every new user;[154]

4.3.2 Temporal factors

The OECD recommends giving due regard to temporal factors, which can differ between merger and non-merger matters. Although the same types of entry factors are considered in merger and non-merger cases, a major difference is that in merger cases, the alleged harm will be prospective. For merger cases - The issue of entry is whether and when a merger will induce enough entry to substantially mitigate the merger's potential anti-competitive effects. Potential entrants who will decide to incur any sunk costs of entry only if they anticipate profitable **post-entry** prices, as distinct from the post-merger (or post anti-competitive conduct) prices that prevail prior to entry. Potential entrants take into account the effect that their own entry will have on the market price. If it is too low for them to enter profitably, then they will not do so. On the other hand, for abuse of dominance/monopolization cases, on the other hand, the harm has already occurred. The issue for

[151] Abrenica, Ramos, and Cleofas III, "Competition Policy and Data Privacy in Social Media and Search Platforms", at 38.

[152] Ibid., at 17.

[153] Ibid., at 18.

[154] Another aspect that contributes to economies of scale for online platforms is the zero pricing model adopted by most incumbents in the online platform space. Zero pricing makes it difficult for new entrants to gain scale, since it will need to match its price levels and then compete on non-price features. ibid., at 23.

entry analysis looks at the past: "Did enough entry occur to cure the harm to competition?"[155]

At the same time, the OECD cautions against taking into account any theoretical, long-term curative effects of entry can take a long time to manifest. For example, there may be overall harm to welfare due to the delay of entry may outweigh any contingent entry that justifies the approval of a merger.[156]

4.3.3 Quantitative Indicators of Barriers to Entry

Sunk costs The OECD likewise warns against Schmalensee's notion that the height of entry barriers can be assessed by measuring the amount of sunk costs that potential entrants would need to incur, as smaller sunk costs lead to lower market prices.[157] Using sunk costs as the primary metric seems to ignore other factor that could deter or prevent entry. For example, it may be the case that entry is simply not possible (at any cost level) because of a patent held by a rival. Likewise, knowing the magnitude of sunk costs alone is of no use unless it can be placed in some kind of relative context to guide policy decisions.[158] For example, sunk costs of 10 million euros may be too high for some entrants but not for others - i.e. it may be "worth it" or justified for some markets but not for others. Entry barrier analysis may have to be case-specific, instead of relying on either a general quantitative guidelines or a single statistical indicator.

The main problem is how to detect sunk costs, determine if they are substantial enough to retard or prevent entry. A lot of mathematical models have been proposed but often involve so many artificial assumptions that are of limited practical value. It is not clear how much weight should be allocated to purely theoretical evidence on entry, such as hypothetical responses to hypothetical

[155] Thus, entry analysis in a non-merger matter is easier because it focuses mainly on whether future entry will be sufficient, as well as what time period will be required. Organisation for Economic Co-operation and Development, "Barriers to Entry", at 39.

[156] Ibid., at 45.

[157] Ibid., at 40.

[158] Ibid., at 40.

price increases. The OECD, based on Salop (1986)[159] suggests practical methods for estimating likelihood of an entry that can prevent anticompetitive price increase:[160]

- **Cost and demand disadvantages** - Includes situations in which entrants have uniformly higher costs than incumbents, as well as those in which entrants must provide discounts to offset perceived differences in quality in comparison with incumbent firms. This includes factors such as restricted access to intellectual property rights/resources, rights and other resources, reputational effects, and brand loyalty. Having data for these parameters will enable an agency to estimate the amount by which an incumbent could raise its price without inducing entry;

- **Time to entry** - The longer entry would take to accomplish, the less likely fear of entry is to deter an entry is to deter an incumbent from raising its prices (Time is necessary not just for entry but to be an effective competitor);

- **Economies of scale** - Scale economies can be measured in terms of Minimum Viable Scale (MVS), the total sales a hypothetical new entrant would need to achieve in order to earn a sufficient rate of return on its invested capital to justify its entry. The smaller the MVS is, the more likely it is that entry will occur that is sufficient to counteract the feared harm to competition from the merger.

Profit Levels Evidence of persistent supra-competitive profits is generally consistent with, but neither necessary nor sufficient for, a finding that barriers are high and that entry is therefore unlikely. Similarly, evidence of a persistent lack of a supra-competitive profits is generally consistent with, but neither necessary nor sufficient for a finding that barriers are low and that entry is therefore easy.[161]

[159] Steven Salop. "Measuring Ease of Entry". In: *Antitrust Bulletin* 31.2 (1986), p. 551. URL: https://doi.org/10.1177/0003603X8603100214.

[160] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 40-41.

[161] Ibid., at 44.

Other Quantitative, Qualitative Factors While the OECD notes that quantitative factors such as Minimum Viable Scale (MVS), or the Herfindahl-Hirschman Index (HHI),[162] can be taken into consideration in determining whether the market is protected by a barrier. As opposed to HHI (which can be computed along one variable - market share) ease of entry analysis is determined by a multitude of interdependent factors, including uncertainty.[163] Baker suggests also looking at other qualitative evidence such as: 1. Documents of industry players; 2. Interviews with executives, employees; 3. Previous instances of entry or attempted entry.[164]

Finally, the OECD suggests looking into repeated instances of past entry under similar conditions as reliable evidence of low barriers.[165] However, previous incidents of actual entry do not necessarily prove that it was easy or competitively meaningful, or that it is likely to take place again. We also need to look at: 1. price effects, 2. the experience of the entrant, 3. viability of past entry strategies.Likewise, although an absence of actual or meaningful entry in the past is consistent with the presence of substantial entry barriers, it does not necessarily prove that significant entry is unlikely in the future.[166]

[162] Herfindahl-Hirschman Index (HHI) - Is a measure of market concentration calculated by summing the squares of the percentage market shares of all firms in an industry. It indicates high concentration (monopoly) at 10,000 and low concentration (perfect competition) at near zero. U.S. Department of Justice. *Herfindahl-Hirschman Index*. Jan. 17, 2024. URL: https://www.justice.gov/atr/herfindahl-hirschman-index (visited on 05/09/2026).

[163] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 47-48.

[164] Jonathan Baker. "The Problem with Baker Hughes and Syufy: On the Role of Entry in Merger Analysis". In: *Antitrust Law Journal* 65 (1997), pp. 353–374. URL: https://digitalcommons.wcl.american.edu/facsch_lawrev/1495.

[165] Organisation for Economic Co-operation and Development, "Barriers to Entry", at 43.

[166] Ibid., at 44.

Chapter 5

Encoding the Semantic Component of the Domain

Based on Chapter 3, we are proceeding from the notion that law and questions of law are largely computable problems.[1] In the last Chapter, we have described the particular domain of law to be encoded - Section A of the OECD Guidelines, with further elaboration of semantics derived from interpretations of EU competition law, as well as competition impact assessments from the PCC.

Facilitating computation of law requires encoding systems for two parallel problems: First to represent, then to analyze these representations(determine relevance, and evaluate for compliance). These appear to be distinct problems and require different encoding systems. The representation of the norms will proceed based on a layered approach (with each layer of the model for the norm manifested from a textual form into a logical account, as well as a programmatic account):

1. The first layer is concerned with the basic semantics of the concepts and relationships that are included in the rule. It will have a logical account based on Description Logic, and a corresponding programmatic account through the Object Web Language.

[1] A computable question is one that can be computed by a sufficiently powerful “Turing machine". See discussion in Chapter 3

2. The second layer is described through the Reified IO logic for its logical account, with the corresponding programmatic account encoded in the Shapes Constraint Language

The first approach aims to capture the semantic content of the law through ontologies, which are used to model the entities and relationships in a domain. The second approach is concerned with representing the normative constraints contained in the law as a set of defeasible inferential statements in deontic logic.[2] This chapter provides an overview of the first approach, with a focus on how they can be applied to the domain of competition law, as discussed in the previous chapter.

As a software artifact, the knowledge base is composed of an ontology layer encoded in the Object Web Language (OWL) and the rules layer encoded in the Shapes Constraint Language (SHACL). The source code is freely accessible online and will be included as an annex to this work. Focused discussion of the knowledge base will be carried out through two forms of related logical notations: A form of Description Logic for the ontology, and Reified I/O Logic for the rules layer. Both logical notations are based on First Order Logic and have been applied to the problem of encoding legal norms. This will allow for a consistent syntax and symbol set for the discussion. The approach also preserves the distinction between two kinds of logical formulae:[3]

1. TBox - the terminological declarative statements, encoded as any formula in DL; and

2. ABox - the assertive contextual statements encoded as flat conjunctions of atomic predications in Reified I/O Logic.

This chapter will be concerned with the TBox, expressed both in terms of Description Logic account as well as a programmatic account in OWL.

[2] It may be possible to combine both the semantic and normative aspects. Both ontologies and inference statements are based on logic and can be arranged into network structures. In the future, machine learning may be used to automatically translate rules into logical formalisms. Meanwhile, the exercise will be undertaken by humans.

[3] Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5690.

5.1 Ontological Representation of Legal Semantics

5.1.1 Scoping

The ontology, its terms, structure, semantics, as well as norms to be encoded, is based on the OECD guidelines as a starting point, but it can be extended to account for other normative frameworks. The programmatic account of the ontology outlined here are usually implemented as fragments of the FOLIO and FIBO ontologies. Whenever possible, the content and structure of the ontologies will be combined and harmonized. For the ontology we are constructing, whenever possible we will avoid the complications of the Open World Assumption.[4] By default (and unless specified) classes with a common superclass will be set as disjoint (i.e. they do not have individuals in common) This convention will also facilitate data validation requirements for later analysis.[5]

As much as practicable the content of the ontology will be restricted to what is necessary to express the semantics in the OECD guidelines. This means that certain classes and relationships will be defined partially, and may not be fully valid based on other domains, such as legal theory or economics.The encoding takes into consideration:

1. The text of the rule in the OECD guidance;
2. Annotations in the guidelines;
3. When applicable, sources from competition law and economics

5.1.2 Definition and Benefits

Law provides a description of a preferred state of the world - a state which can be made legible as a configuration of entities and relationships. The entities are the actors, transactions, and objects

[4]The Open World Assumption - is the assumption that the knowledge in the ontology is incomplete, and there might be other individuals or relationships not yet explicitly stated. Thus if something is not explicitly stated in an ontology - it is not assumed to be false; it is simply unknown.

[5]Michael De Bellis. *A Practical Guide to Building OWL Ontologies*. Oct. 8, 2021. URL: https://www.michaeldebellis.com/post/new-protege-pizza-tutorial (visited on 01/31/2024), at 76.

that are the subjects of the law. The relationships are the connections between these entities, and the attributes that describe them. This aspect of the law can be encoded as an ontology. An **ontology** is a formal, explicit description of concepts that are part of a domain.[6]. In computer science an ontology is “a formal description of all the entities of a domain and the relations existing between these entries”. Part of the "symbolic" approach to artificial intelligence - which requires encoding knowledge into a structure amenable to processing by computers.[7]

It consists of: 1. **classes** that represent concepts; 2. **properties** that describe features of these concepts, including their relationship with each other; and 3. **restrictions** to the way these classes and attributes are defined.[8] An ontology of classes, along with specific instances of these classes, constitute a **knowledge base**, although as a practical matter there can be little to distinguish this from an ontology.[9] Ontologies can be used to make web pages (or other electronic resources) more “understandable" to electronic agents. Many disciplines are developing standardized ontologies used by experts to encode, annotate, and share knowledge in their respective fields,providing a common vocabulary researchers and a source of machine-readable definitions.[10] Noy (2001) suggests that for extensive domains of knowledge, ontologies can provide the following benefits:

1. *Sharing and collaboration* Experts and practitioners can represent their shared understanding.

2. *Enabling reuse* Users can build on existing ontologies - extending or refining them as needed.

[6]Natalya F Noy and Deborah L McGuinness. “Ontology Development 101: A Guide to Creating Your First Ontology”. In: *Stanford Medical Informatics Technical Report* (SMI-2001-0880 Mar. 2001). URL: http://www.ksl.stanford.edu/people/dlm/papers/ontology-tutorial-noy-mcguinness-abstract.html, at 3. The term ontology originally referred to a branch of philosophy concerned with the study of being. It was borrowed by computer science to refer to the formal definition of objects in a domain, and the relationships between these objects. See Lamy Jean-Baptiste. *Ontologies with Python: Programming OWL 2.0 Ontologies with Python and Owlready2*. Berkeley, CA: Apress, 2021. ISBN: 978-1-4842-6551-2 978-1-4842-6552-9. DOI: 10.1007/978-1-4842-6552-9. URL: http://link.springer.com/10.1007/978-1-4842-6552-9 (visited on 04/03/2024), at §3, p. 61.

[7]Jean-Baptiste, *Ontologies with Python*, at § 1.2, p. 19.

[8]De Bellis, *A Practical Guide to Building OWL Ontologies*, at 6.

[9]Noy and McGuinness, “Ontology Development 101: A Guide to Creating Your First Ontology”.

[10]For medicine, for example, there is SNOMED (Price and Spackman, 2000) and the Unified Medical Language System (Humphrey and Lindberg, 1993); For describing products and services for the purpose of trade regulation, see the United Nations Standard Products and Services Code(UNSPC), at https://www.unspsc.org/

3. *Making assumptions explicit* Assumptions can become explicit in the design of an ontology, making it easier to question and resolve them as necessary.

4. *Separating domain knowledge from operational knowledge* We can analyze a class of concepts in the abstract, independent of particular instances.

5. *Analyzing domain knowledge* Once a representation is available, it can be subjected to formal analysis.

Managing and retrieving data from ontologies is more efficient and cost-effective compared to Large Language Models (LLMs). To make corrections, one simply needs to identify and modify the specific entity and attribute. This approach is more appropriate for making precise factual determinations where accuracy is prioritized over expressiveness. The use of ontologies is also more transparent and interpretable compared to LLMs. The structure of the ontology can be visualized and understood by humans, and the reasoning process can be traced and explained. This is important for legal applications, where the reasoning process must be transparent and understandable to the parties involved.

5.1.3 Ontologies as a Description Logic Account

From Propositional to Description Logic An ontology can also be seen as a set of axioms, written in Description Logic (which formalizes the definitions of entities into logical axioms).[11] Prior to covering the actual computational encoding, it will be necessary to explain the underlying logical account DL. Most formalized logic is built on the basic layer of propositional logic. Propositional logic (or propositional calculus) is concerned with representing valid arguments and logical truths that can be expressed through "atomic" propositions or compound "molecular" propositions (formed from the former through extensional, truth-functional operators such as "and", "not") The concern is whether or not the argument from "p and q" to "p " is valid. It is not concerned with the content and

[11] Jean-Baptiste, *Ontologies with Python*, at § 3.1, p. 62.

the internal structure of the propositions themselves.[12] Propositional logic requires: 1. Propositional letters or variables: e.g p, q, r and so on. Range over propositions (we assume that there is an unlimited number of propositions and variables); 2. Propositional connectives;[13] 3. Auxiliary signs: ( ) - used to set order of operation within combinations of formulae. Propositional logic is the basis of syllogistic statements, i.e., where the premises of the argument support its conclusions. These arguments are said to be valid in the sense that the premises imply their conclusions.[14] Predicate logic (or predicate calculus) extends propositional logic, representing valid arguments and logical truths that depend on the internal structure of propositions.[15] Predicate logic is a formal system used to represent statements and arguments[16]. It goes beyond propositional logic by allowing us to talk about objects and their properties - A "predicate" is a statement about an object. E.g. "is tall" is a predicate. Description logic is a further subset of predicate logic, extending it through by the introduction quantifiers, variables, and predicates. It is not one logic, but can refer to a whole family of knowledge description languages.[17] A decideable fragment of DL called $\mathcal{SROIQ}$ is the basis for the Object Web Language and will be used to notate the ontology formulae in this section.[18]

The semantics (or meanings) of the terms of DL, concepts can be interpreted in terms of set theory:[19]

1. A concept (class) is a set of individuals;

[12] Haack, "On Logic in the Law", at 11.

[13] This includes 1. monadic connectives such as ~ (negation);2. dyadic connectives like ∧ (conjunction), ∨ (disjunction),→ (conditional), ↔ (biconditional); Navarro and Rodríguez, *Deontic Logic and Legal Systems*.

[14] John Heil. *First-Order Logic - A Concise Introduction*. Indianapolis, Indiana: Hackett Publishing Company, 2021, at 8.

[15] This analysis of internal structure is carried out through quantifiers (i.e. "for all x", "for some x"), singular terms or names (e.g. "a", "b", etc) and predicates, i.e. "for all x, if Fx then Gx" Haack, "On Logic in the Law", at 11.

[16] A statement (or proposition) - is a declarative sentence that can either be true or false. In predicate logic, statements can have predicates, variables, quantifiers, and logical connectives. On the other hand, an argument is a collection of statements/propositions, where one or more of them (the premises) are intended to provide support for another statement/proposition (the conclusion). Arguments are concerned about reasoning and drawing inferences (based on the claim that the conclusion follows logically from the premises)

[17] Markus Krötzsch, Frantisek Simancik, and Ian Horrocks. "A Description Logic Primer". In: *IEEE Intelligent Systems* 29.1 (2014), pp. 12–19. DOI: 10.48550/arXiv.1201.4089. arXiv: 1201.4089 [cs]. URL: http://arxiv.org/abs/1201.4089 (visited on 06/20/2025), at 1.

[18] Ibid., at 1.

[19] Daniele Nardi and Ronald J. Brachman. "An Introduction to Description Logics". In: *The Description Logic Handbook*. Cambridge University Press, 2007, pp. 1–43, at 8.

2. Roles (attributes or properties) are sets of pair of individuals;

3. Domain of interpretation - can be arbitrarily chosen, and can be infinite (if so, then the open world assumption holds)

Furthermore, we can define atomic concepts as subsets of the domain of interpretation.The semantics of other constructs can be specified by delimiting the set of individuals denoted by each construct (e.g. the concept $C \sqcap P$) is the set of individuals obtained by intersecting the sets of individuals denoted by C and D.[20]

In the case of entities and relationships related to the knowledge domain of competition impact assessment, we can represent atomic concepts (also called classes and subclasses)[21](such as $Person$, $JuridicalEntity$), and define other concepts in terms of these atomic concepts, e.g. A $Natural\_Person$ should be any person that is not a juridical entity ($Person \sqcap \neg JuridicalEntity$). In DL, nouns in the legal text can be mapped into atomic concepts, while verbs and descriptors into atomic roles, with subsequent terms defined by combining these components into through logical operators. For example, a $Buyer$ is defined as an $Actor$ that is also a $Role\_in\_a\_Legal\_Matter$: $Buyer \sqsubseteq Actor \sqcap Role\_in\_a\_Legal\_Matter$. This can also be further defined in terms of what it is not (A $Buyer$ cannot be a $Seller$): $Buyer \sqsubseteq Actor \sqcap \neg Seller$. DL also supports atomic roles (called properties in OWL) that can define the relationships between concepts using operators such as intersection, union, and complement (interpreted as set operations) we can describe concepts in terms of other concepts. For example, if the knowledge domain for example requires that every juridical entity should have a director, then an ontology with a `Juridical_Entity` class is bound to a `Director` class through the $hasDirector$ role: $Juridical\_Entity \equiv \exists hasDirector.Director$. That is, one definition of a juridical entity is that it belongs to the set of objects that is related to the set of objects called $Director$ through the $hasDirector$ role. We can further qualify the relationship by

[20]Thus $\forall R \cdot C$ is the set of individuals that are in relationship R with individuals belonging to the set denoted by C. ibid., at 8.

[21]Ibid., at 8.

imposing domain restriction to the role. For example, we can specify that only natural persons who are adults, or $Adult \equiv Natural\_Person \sqcap hasAge.(\geq 18)$ can serve as directors. Role restrictions, such as those pertaining to cardinality, can help us define the requirement that a juridical entity should at least have 5 directors, for example: $Juridical\_Entity \equiv \geq 5 hasDirector.Director$

Enabling analysis and intelligence Description logic not only allows us to capture facts about the world in an unambiguous manner, it also enables having the ability to find implicit consequences from the explicitly represented knowledge -e.g. through following inductive or deductive chains from the given ontological posits.[22] This ability can be supported and visualized, by thinking of the knowledge encoded in DL as a network structure, where nodes correspond to classes or individuals, while the links or arcs connecting them are the relationships and attributes.[23] The analysis and discovery enabled by DL encoding can be performed through the standard logical notation, with logical operations performed with pen and paper. However, although the rest of this dissertation may use DL notation to refer to specific objects and relationships, the analysis and visualization will be performed using computational tools built around the Semantic Web.

5.1.4 Ontologies and the Semantic Web

Since every modern computer language is Turing complete (i.e. it can fully implement a Turing machine), these programming languages are capable of computing legal questions. The only constraints will be time, memory, and computing power. Andersson (2014) asserts that most software tools (general purposes, modern languages) are overkill for implementing the requirements of a computational law system. It would be more efficient (cost-benefit wise) to develop and use domain-specific languages for computational law.[24] However, it is very difficult to come up with domain specific languages specific to law - this may be a function of few lawyers knowing how to program, and few programmers understanding law. For this encoding exercise, the programmatic

[22] Nardi and Brachman, "An Introduction to Description Logics", at 2.
[23] Ibid.
[24] See Andersson, "Computational Law: Law That Works Like Software", at 21.

account for both the TBox and ABox will employ Semantic Web Technologies.

The Semantic Web is a set of layered technology standards (similar to the technology stack of the World Wide Web) that extends the World Wide Web to make web content understandable to machines.[25]It is an effort to apply the open the open, shareable, connected nature of document-based web to data: I.e. through a common framework that allows data to be shared and reused across applications, enterprise, and community boundaries, for automated processing by tools, including revealing possible new relationships among pieces of data.[26] All semantic web technologies are built on the Resource Description Format (RDF), which employs a graph model for representing objects. Thus - the entities, attributes that we model in OWL, as well as the constraints modeled in SHACL (both of which are extensions of the RDF standard) can be viewed as graph structures.[27]

The relationship between computational law (at least the symbolic approach) is complementary/symbiotic. Semantic web technologies provide the essential digital infrastructure standard data formats, and ontological vocabularies that can (as demonstrated by this work) - help translate unstructured text into machine executable logic. Semantic web standards provide a medium for construction and sharing of models that can help practitioners communicate, make predictions, and mediate among multiple viewpoints (by being able to represent and surface commonalities).[28]

An ontology can be formally expressed in a computer language. This work will use the Web Ontology Language (OWL) to express ontologies. The choice is largely based on the OWL's broad adoption, and the availability of supporting software and documentation.OWL is a standard that is maintained by the World Wide Web Consortium (W3C), the organization that sets standards

[25]While the World Wide Web is made up of documents linked to each other, the Semantic Web is made up of referers to the things in the world called resources. Semantic web resources can be anything in a domain of discourse Dean Allemang, James Hendler, and Fabien Gandon. *Semantic Web for the Working Ontologist - Effective Modeling for Linked Data, RDFS, and OWL*. Association for Computing Machinery, 2020. ISBN: 978-1-4503-7615-0, at 27.

[26]World Wide Web Consortium. *W3C Semantic Web FAQ*. 2009. URL: https://www.w3.org/RDF/FAQ (visited on 04/20/2026).

[27]Ibid.

[28]Allemang, Hendler, and Gandon, *Semantic Web for the Working Ontologist - Effective Modeling for Linked Data, RDFS, and OWL*, at 13.

for the web. It is used to represent knowledge in a way that is machine-readable and can be processed by computers. OWL is based on the Resource Description Framework (RDF), a standard for representing information on the web. RDF is used to represent information in the form of triples, which consist of a subject, a predicate, and an object. OWL extends RDF by providing a way to represent classes, properties, and relationships between classes and properties. It also provides a way to represent restrictions on classes and properties, such as cardinality constraints and value constraints. OWL is used in a wide range of applications, including the Semantic Web, data integration, and knowledge representation. It is a powerful language that can be used to represent complex knowledge in a structured and formal way. OWL is a language that is based on Description Logic, a subset of predicate logic that is used to represent knowledge in a structured and formal way.[29]

Description Logic (DL) is a term for a family of knowledge representation languages.

For prototyping and visualization purposes, the author will use the Protégé ontology editor, which is a widely used tool for creating and editing ontologies in OWL.[30] Protégé is an open source program that supports ontology design and construction through a variety of semantic web standards and can be extended to perform reasoning on ontologies.[31] Although intended for a wide range of applications, Protégé is commonly used in the computational medicine community for notable projects such as the Open Biological and Biomedical Ontology Foundry[32] as well as the World Health Organization's International Classification of Diseases.[33]

[29] See Noy and McGuinness, "Ontology Development 101: A Guide to Creating Your First Ontology", at 3.

[30] See Mark A. Musen. "The protégé project: a look back and a look forward". In: *AI Matters* 1.4 (2015), pp. 4–12. DOI: 10.1145/2757001.2757003. URL: https://doi.org/10.1145/2757001.2757003.

[31] Stanford Division of Computational Medicine. *Protégé (Homepage)*. 2016. URL: https://protege.stanford.edu/ (visited on 05/14/2026).

[32] See OBO Technical Working Group. *OBO Foundry*. 2024. URL: https://obofoundry.org/ (visited on 05/14/2026).

[33] World Health Organization. *International Classification of Diseases (ICD)*. 2026. URL: https://icd.who.int/browse/2026-01/mms/en (visited on 05/14/2026).

5.1.5 Ontology components

Classes are the primary focus and building blocks of an ontology. These describe concepts in a domain.[34] Since we are concerned with modelling entities that interact with each other and the law, our ontology can have a `Person` class that represents the legal definition of a person - an individual or entity that has the capacity to enter into legal relations. A class can have **subclasses** that represent more specific concepts.[35] For example, the `Person` class can have subclasses such as `Natural_Person` to represent a human individual and `Juridical_Entity`, such as a corporation. Individuals (or *instances* of these classes) are the actual objects in the domain of interest.[36] For example, the `Natural_Person` class can have instances such as `Alice` and `Bob`.

Properties and inheritance describe the attributes of and relationships among classes and instances. The class definition of the `Person` class can have `has_name` property that describes the name of a person, which can be provided for an instance of that class. Properties can also be used to describe the relationships between classes. For example, the `Person` class can have a `has_child` property that describes the relationship between a parent and a child. The `has_child` property can be used to connect a `Natural_Person` instance to another `Natural_Person` instance that is their child. Properties can also have restrictions that define the cardinality of the property, the value of the property, or the relationship between the property and other properties. For example, the `has_child` property can have a restriction that specifies that a child can have at most two parents. Subclasses inherit the properties of their parent classes, and can have additional properties that are specific to them.[37]

[34] Noy and McGuinness, "Ontology Development 101: A Guide to Creating Your First Ontology", at 3.

[35] Ibid., at 3.

[36] See De Bellis, *A Practical Guide to Building OWL Ontologies*, at 7. At the same time, classes can be thought of as sets that contain individuals.

[37] See ibid., at 7.

5.2 Ontology construction

There is no one "right" methodology for constructing an ontology. An ontology, at its core is only a model for reality, and there can be a multiplicity of design choices as to what parts of reality are deemed relevant, based on the problem we want to solve. Noy(2001) proposes an iterative approach: With a rough, initial pass, filling details along the way. It is a question of what is most appropriate for the applications in mind and the developments anticipated for the ontology. There should at least be a sense of isomorphism, or closeness, between an ontology and the common understanding of the domain.[38] This can be achieved by reflecting on the statements that describe the domain. The nouns correspond to the classes/instances, while the verbs and adjectives correspond to the attributes.

The Reference Ontology shall be designed based on the following process outline in Noy(2001), with some details provided by DeBellis (2021):

1. Determine the domain and scope of the ontology
2. Consider reusing existing ontologies
3. Enumerate important terms
4. Define the classes and class hierarchy
5. Define the internal structure of classes
6. Define the restrictions of attributes

This tour of the design process will also serve as an opportunity to describe how ontologies can model the semantic content of the relevant competition law, as well as some of the early design decisions taken.

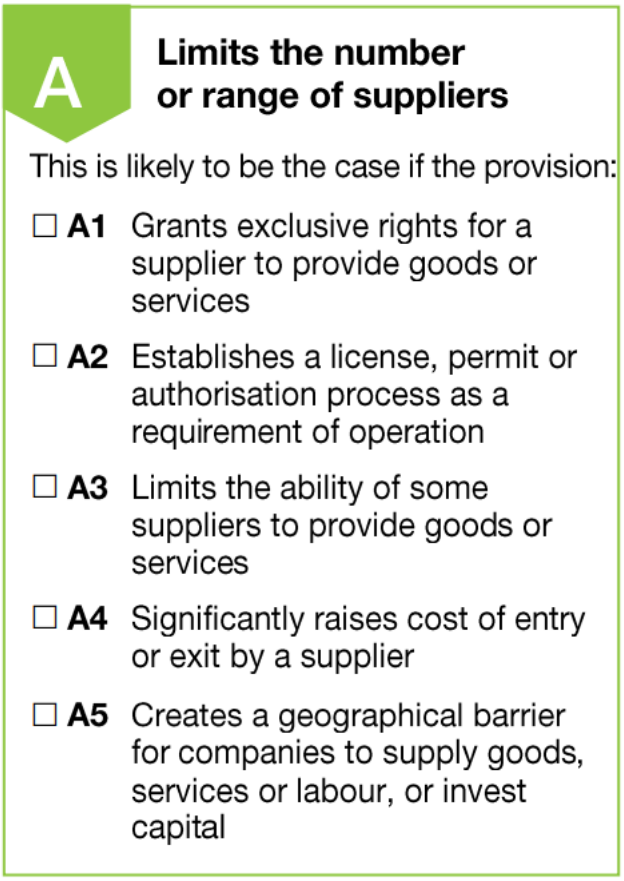
[38] Noy and McGuinness, "Ontology Development 101: A Guide to Creating Your First Ontology", at 4.

5.2.1 STEP 1: Determine the domain and scope of the ontology

The first step requires us to specify the domain of interest, as well as the contemplated uses of the ontology.This study is concerned with several domains, each of which can be modelled through separate ontologies: 1. The idealized configuration of entities and transactions in a competitive market, as described by the OECD threshold tests (the ''Reference Ontology"); and 2. The entities and transactions in the digital payments market in the Philippines, as described by the relevant laws (the ''Evaluated Ontology"); As discussed in the previous chapter, this work will focus on the OECD Guidelines (with additional semantics provided by EU competition law, as well as instances of PCC competition impact assessments)since it has become the *ad hoc* basis of the Philippine's competition impact assessment regime. It is also the most comprehensive and most updated resource of this type available to the public. Other similar guidelines, such as those issued by the International Competition Network, the Asia Pacific Economic Cooperation, and the UK's Competition and Markets Authority will be used to supplement our understanding of the norms applicable to competition impact assessment. The primary purpose of the ontology is to enable the evaluation of laws governing a particular sector for competition effects. For this chapter, we will use the first OECD threshold test standard as an example. The OECD tests for the following competition concerns:

For this demonstration of the design process we are only concerned with norms contained in subsection A, which provides norms as to the preferred structure of a competitive market - e.g. no single or group of participants dominant, and with low or no barriers to entry. Note that although the header for Section A by itself is not a threshold test, and its general normative requirement (i.e., that it not "limits the number or range of suppliers"), it is considered part of the domain since it may still provide information as to required classes and properties.

Figure 5.1: Section A, OECD Threshold Test Checklist



5.2.2 STEP 2: Consider reusing existing ontologies

The knowledge base can be also be based on existing ontologies that have already been developed for some knowledge domains or specific activities. For example,since the Evaluated Ontology will be based on the financial sector, we can use the Financial Industry Business Ontology (FIBO), a knowledge graph that models the entities and transactions in the financial sector.[39] It is a standard that is already being used by financial institutions, regulators, and other stakeholders. For concepts related to law, we may derive from the design of Legal Knowledge Interchange Format (LKIF).[40]

[39] See EDM Council. *The Financial Industry Business Ontology*. FIBO. URL: https://spec.edmcouncil.org/fibo/ (visited on 01/18/2024).

[40] Oasis Open. *LegalRuleML Core Specification Version 1.0*. Aug. 30, 2021. URL: http://docs.oasis-open.org/legalruleml/legalruleml-core-spec/v1.0/legalruleml-core-spec-v1.0.html (visited on 10/06/2023), See also See https://www.gecad.isep.ipp.pt/ieso/contract/v1.0.0/#description for a basic contract ontology.

Modeling the OECD Competition Impact Assessment Guidelines into an ontology that relies on FIBO and LKIF semantics, we must bridge the distinct domains that these ontologies govern. Whenever appropriate, we can use these ontologies directly, or design our ontology to be compatible with them:

1. LKIF provides the semantics for legal and regulatory frameworks. The Reference Ontology uses it to ground concepts like `Regulation`, `AlternativeOption`, and the `CompetitionAssessment` process itself (treating the assessment as an evaluative legal action and the regulation as a legal norm).

2. FIBO provides the semantics for corporate, financial, and market structures. We can use it to ground economic concepts like `Market`, `Supplier`, `Consumer`, and `Incumbent`.

Thus, in addition to the basic RDF and OWL semantics, the ontology refers to the FIBO and LKIF namespaces, so that we can use their classes and properties directly from within the Reference Ontology:

Listing 5.1: Prefix Declarations for the Reference Ontology

```
@prefix : <http://www.semanticweb.org/emersonbanez/ontologies/2026/4/
   oed_cia_ontology/> .
@prefix owl: <http://www.w3.org/2002/07/owl#> .
@prefix rdf: <http://www.w3.org/1999/02/22-rdf-syntax-ns#> .
@prefix rdfs: <http://www.w3.org/2000/01/rdf-schema#> .

# External Ontology Prefixes (External Knowledge)
@prefix fibo-fbc-fct-mkt: <https://spec.edmcouncil.org/fibo/ontology/
   FBC/FunctionalEntities/Markets/> .
@prefix fibo-fnd-org-org: <https://spec.edmcouncil.org/fibo/ontology/
   FND/Organizations/Organizations/> .
```

```
@prefix lkif-norm: <http://www.estrellaproject.org/lkif-core/norm.owl#>
    .
@prefix lkif-action: <http://www.estrellaproject.org/lkif-core/action.
   owl#> .
```

The Legal Knowledge Interchange (LKIF) Core ontology is basic library of primitive legal concepts, meant to be re-used by end-users (citizens, lawyers, legal scholars) who can provide the concepts it contains further definition. Thus, for semantics related to legal concepts, the ontology also adopts the encoding in LKIF. This includes an appropriate conception of an actor in the legal domain, as well as what constitutes a legally valid norm. The top layer of LKIF-core is relied upon to provide the framework for instantiating legal concepts (which can be abstract) and legal subjects (both physical or abstract) in a shared conception of universe that is still subject to constraints of location, time, parthood, and change.[41]

Agents in LKIF LKIF adopts the premise that legal reasoning is based on the behavior of rational agents that can be effectively influenced by the law. These agents are represented by the class of `Agents` that, while in a particular `Role`, perform `Actions`. Agents are capable of maintaining internal mental states (through `Intention` and `Belief`) as well as communication with other agents through `Expressions`.Agents can be subdivided into individual agents called `Persons` (e.g. Pope Leo XIV, or Jean Tirole). and `Organizations` which are aggregates of persons (or other organizations) - to which we can subscribe a unified internal mental state, and to which we can attribute actions.[42] In LKIF, any agent is a vector of an intended outcome of an action. That is, all actions are intentional. `Actions` are `Processes` in that they result through some causal necessity, in some `Change`.[43] An `Agent`'s subclass can include `Person` - which can be a `Natural_Person` or a `Legal_Person`. The LKIF concept of `Actor` contemplates that it can take on a particular role in a

[41] Rinke Hoekstra et al. "The LKIF Core Ontology of Basic Legal Concepts". In: *Proceedings of LOAIT 07*. II Workshop on Legal Ontologies and Artificial Intelligence Techniques. 2007, p. 43–63, at 48.

[42] Ibid., at 53.

[43] Ibid., at 52.

legal matter or transaction. For our purposes, such role can be as `Seller` or `Buyer` in a market. The LKIF connects `Agents` and `Actions` through the concept of a `Role`. A role is not just a classification scheme (i.e some actions relate to only some rules) Roles also convey normative information, in the sense that they can specify what the standard is for certain behaviors. Roles can thus also be the basis of how other agents are expected to behave. The LKIF connects `Agents` and `Actions` through the concept of a `Role`. A role is not just a classification scheme (i.e some actions relate to only some rules) Roles also convey normative information, in the sense that they can specify what the standard is for certain behaviors. Roles can thus also be the basis of how other agents are expected to behave.[44]

Legal Norms in LKIF LKIF also contains a selection of pre-built classes that correspond to concepts in the legal domain - legal agents, actions, rights, and powers.[45]A norm applies to a situation, connecting it to it certain qualifications. I.e. A situation can be obligatory, allowed, or prohibited.[46]

5.2.3 STEP 3: Enumerate and organize important terms

We next proceed to listing the important terms that the ontology needs to describe and explain, as well as their relevant attributes. Attributes can include **property attributes**, which can qualify classes (i.e. what they are “like"), while **functional attributes** can describe what the classes can do, or what can be done to them.[47] As an initial step to The rule of thumb is to consider the nouns of statements as the classes of the ontology, while adjectives and verbs can be considered as the properties. For the Reference Ontology, we can start with the following categorization of nouns/classes:

- **The Assessment Process:** The `CompetitionAssessment` class evaluates an instance of

[44]Ibid., at 54.
[45]Ibid., at 56.
[46]Ibid., at 56-57.
[47]Noy and McGuinness, “Ontology Development 101: A Guide to Creating Your First Ontology”, at 6.

`Regulation` using a `CompetitionChecklist`. If a regulation poses a significant risk to competition, the assessment requires proposing a less restrictive `AlternativeOption`.

- **Barriers to Entry:** Market impediments are formalized into `StructuralBarrier` (exogenous industry characteristics like sunk costs), `StrategicBarrier` (deliberate entry-deterring behavior by incumbents), and `LegalBarrier` (government-imposed regulations or intellectual property rights). The ontology evaluates potential entry based on whether it is `Timely` (often capable of having an impact within two years), `Likely` (expected to be profitable), and `Sufficient` (adequate in scale to deter anti-competitive effects)

- **Abuse of Dominance:** The framework captures anticompetitive strategies deployed by dominant incumbents, including `RefusalToDeal`, `MarginSqueeze`, `ExclusiveDealing`, and `TyingAndBundling`.

A partial list of nouns and verbs in the OECD Guidelines that can be encoded as classes and properties in the Reference Ontology is provided in Table 5.1 below:

Table 5.1: Example terms for the ontology

Nouns (Classes)	**Verbs or Qualifiers (Attributes)**
Competition Assessment Law (or Regulation) Alternative Option Limitation (on Supplier) Exclusive Right License (or Permit) Market Supplier Consumer	evaluates (a Regulation) imposes (a Barrier) screens (for Limitation) participates (in a Market) is dominant deters (Entrant)

The ontology designer should also take note of any term that may be in the statement being modelled, but are nevertheless implied by the other terms. For example, since the standards mention a `Supplier`, it can be inferred even at this point that we need to model the ultimate recipient of the goods and services supplied - a `Consumer`. Both `Supplier` and `Consumer` are subclasses of `Person`,

which we will also need to define and elaborate later on. Finally, since the standards in the threshold test are meant to apply to laws - hence the need for a `Law` class. The `State` class is also implied, as the standards assume that there is a state that is enacting and enforcing the law.[48]

5.2.4 STEP 4: Define the classes and class hierarchy

Several approaches are open to determining the classes and their place in the hierarchy (i.e. the subclass-superclass relationship). There is the **top-down approach** which is to start with the most general concepts, and then proceed to the more specific cases. Alternatively, one can also take a **bottom-up approach**, which means to start with defining the most specific classes, then determine if these can be grouped into general concepts (i.e. generate common superclasses). The more realistic approach is a combination of both, i.e. define the salient concepts and then generalize or specialize as needed. No method is best - it would depend on the circumstances surrounding the modeling, i.e. if a general view is available, if data is granular enough to describe specific cases.[49] To determine which terms can be classes or subclasses, a good rule of thumb is that objects that are capable of independent existence (rather than descriptions of other objects) can be the principal classes in a class hierarchy.[50] Class definitions automatically give rise to a class hierarchy, since most classes are defined as a subclass of some other class (e.g. a `PatentRegime` can be considered as a type of `LegalBarrier`, which in turn is a subclass of `BarrierToEntry`). The class hierarchy, as constructed in Protégé can be visualized as shown in the following figure:

Note that in OWL, all classes are subclasses of a root class called `owl:Thing`, the class that represents the set containing all individuals. All empty ontologies still contain one class called `owl:Thing`.[51]

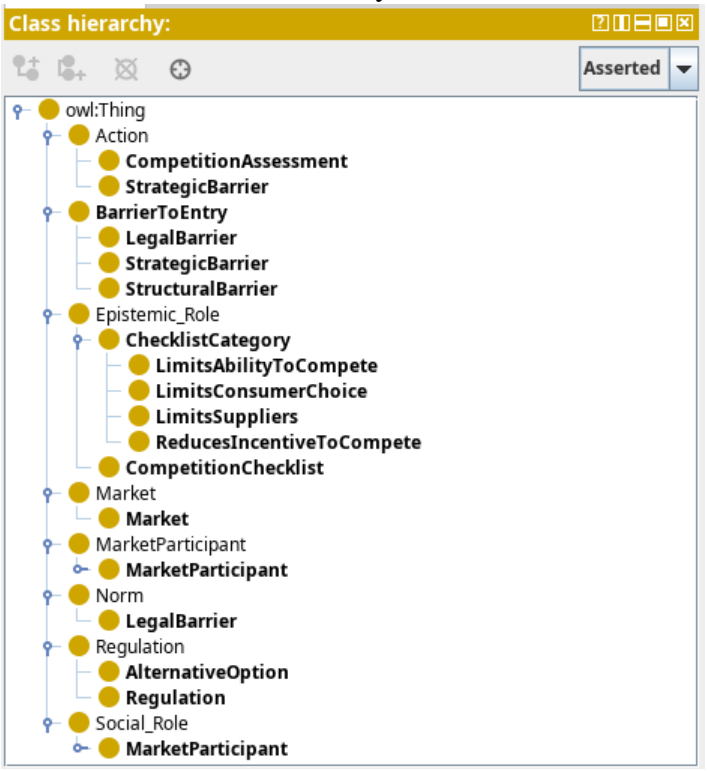
[48] Although the text of the OECD tests refers to some concepts in the plural (e.g. “Goods"), the naming convention will use the singular form. Classes represent sets and can contain multiple instances. Thus, it is not necessary to define singular forms of classes as subclasses.

[49] Noy and McGuinness, “Ontology Development 101: A Guide to Creating Your First Ontology”, at 6-7.

[50] Ibid., at 7-8.

[51] De Bellis, *A Practical Guide to Building OWL Ontologies*.

Figure 5.2: Partial class hierarchy for the Reference Ontology



Since we are concerned with Section A, the Reference Ontology models a situation where a regulation is screened to determine if it limits the number or range of suppliers. The provided OWL ontology models the four checklist categories as direct sub-classes of the `:ChecklistCategory` class, serving as the core screening mechanism used during a `:CompetitionAssessment` (See code listing below). The ontology uses the `:usesChecklist` and `:screensFor` object properties to evaluate whether a `:Regulation` falls into the categories under Section A. Particularly, whether the law limits the number or range of suppliers (`:LimitsSuppliers`) The ontology uses this category to screen for provisions that restrict market participation. According to the OECD Guidelines, this includes regulations that grant exclusive rights to a supplier, establish a license, permit, or authorization process as an operational requirement, or limit the ability of certain suppliers to provide goods or services. It also screens for rules that significantly raise the costs of entry or exit, or create geographical barriers for companies attempting to supply goods, services, or capital. The ontology enables a systematic evaluation of whether a proposed or existing regulation creates barriers to

entry, limits competitive actions, or restricts consumer freedom, which ultimately determines if a less restrictive `:AlternativeOption` must be proposed.

Listing 5.2: Class Definition for Regulation and Alternative Option

```
:CompetitionAssessment rdf:type owl:Class ;
    rdfs:subClassOf action:Action ;
    rdfs:label "Competition Assessment" .

:CompetitionChecklist rdf:type owl:Class ;
    rdfs:subClassOf role:Epistemic_Role ;
    rdfs:label "Competition Checklist" ;
    rdfs:subClassOf [
        rdf:type owl:Class ;
        owl:intersectionOf (
            [ rdf:type owl:Restriction ; owl:onProperty :screensFor ;
              owl:someValuesFrom :LimitsSuppliers ]
            [ rdf:type owl:Restriction ; owl:onProperty :screensFor ;
              owl:someValuesFrom :LimitsAbilityToCompete ]
            [ rdf:type owl:Restriction ; owl:onProperty :screensFor ;
              owl:someValuesFrom :ReducesIncentiveToCompete ]
            [ rdf:type owl:Restriction ; owl:onProperty :screensFor ;
              owl:someValuesFrom :LimitsConsumerChoice ]
        )
    ] .
```

LKIF provides the semantics for legal and regulatory frameworks. The Reference Ontology uses it to ground concepts like `Regulation`, `AlternativeOption`, and the `CompetitionAssessment` process itself (treating the assessment as an evaluative legal action and the regulation as a legal norm).

The :Regulation and :AlternativeOption classes are rooted in **lkif-norm** schemas representing legislative acts and normative statements (see code listing below). Furthermore, the :CompetitionAssessment is mapped as an lkif-action:Observation, capturing the investigative nature of policy review.

Listing 5.3: Class Definition for Competition Assessment

```
:Regulation rdf:type owl:Class ;
    rdfs:subClassOf norm:Regulation ;
    rdfs:label "Regulation" ;
    rdfs:comment "A law, policy, or public intervention being evaluated
       . Under LKIF semantics, it is a legal document bearing one or
       more norms uttered by a legislative body." .

:AlternativeOption rdf:type owl:Class ;
    rdfs:subClassOf norm:Regulation ;
    rdfs:label "Alternative Option" ;
    rdfs:comment "A less restrictive policy alternative identified
       during the assessment that achieves the same government
       objective without unduly harming competition." .
```

FIBO provides the semantics for corporate, financial, and market structures. The Reference Ontology can use it to ground economic concepts like Market, Supplier, Consumer, and Incumbent. Particularly, the :Market and :MarketParticipant (and its sub-classes :Supplier, :Consumer, :Incumbent, and :Entrant) are grounded in the **fibo-fbc-fct-mkt** (Financial Business and Commerce - Markets) domain, linking the OECD's focus on supply and demand constraints directly to standardized financial enterprise architecture.

Cross-Domain Concepts `:LegalBarrier` inherits from both `:BarrierToEntry` and `lkif-norm:Norm`, representing the exact intersection of legal norms (from LKIF) generating economic market constraints (from FIBO).

Listing 5.4: Class Definition for Barrier to Entry

```
:BarrierToEntry rdf:type owl:Class ;
rdfs:label "Barrier to Entry" .

:LegalBarrier rdf:type owl:Class ;
    rdfs:subClassOf :BarrierToEntry , lkif-norm:Norm ;
    rdfs:label "Legal Barrier" ;
    rdfs:comment "Legal barriers arise from legislation, sector
       regulations, or technical regulations." .
```

An exhaustive enumeration of all classes and their relationship with all other classes will be beyond the scope of this section - this is nevertheless transparent through the code, and can be explored through Protégé. A partial view of the Reference Ontology's classes and their hierarchical relationships can be seen in Figure 5.2 below. The Reference Ontology is organized along the following core branches:

1. **Assessment & Legal Procedural Elements:** This branch models the evaluation process itself. `Regulation` and `AlternativeOption` are subclasses of `norm:Regulation`, which LKIF-Core defines as a document bearing one or more norms uttered by a legislative body. The `CompetitionAssessment` is an `action:Action`, which represents a deliberate, intentional change brought about by an evaluating agent playing a role.

2. **Checklist Categories & Dominance Indicators:** Derived from the OECD toolkit, these classes define the analytical screening mechanisms. Classes such as `CompetitionChecklist`, `ChecklistCategory` (including `LimitsSuppliers` and its specific sub-categories

like `GrantsExclusiveRights`), and `DominanceIndicator` (such as `MarketShare` and `LackOfSubstitutability`) are modeled as subclasses of `role:Epistemic_Role`. In LKIF, an epistemic role represents an entity used specifically within a reasoning, inference, or assessment process.

3. **Market Actors:** This branch captures the economic environment. `MarketParticipant` (and its subclasses `Supplier`, `Consumer`, `Entrant`, and `DominantSupplier`) inherits from both FIBO and LKIF's `role:Social_Role`, defining actors as entities operating in social activities that carry expectations of default behavior.

4. **Barriers to Entry:** Entry impediments are categorized into `StructuralBarrier`, `StrategicBarrier`, and `LegalBarrier`. A `LegalBarrier` intersects with `norm:Norm` (a legal qualification that allows or disallows behavior), while a `StrategicBarrier` intersects with `action:Action` to reflect the deliberate, tactical, and exclusionary behavior of incumbent firms.

Class hierarchies show how concepts are related. They use terms like "is-a" or "kind-of" to show these connections. When one class is a subclass of another, it means the subclass represents a more specific type of the general concept represented by the main class.[52] A subclass relationship is transitive, i.e. "If B is a subclass of A and is a subclass of B, the C is a subclass of A".[53]. It may also be useful to determine at this point which classes are *disjoint*,i.e., that no individual can be an instance of more than one of those classes.[54] In our example, `StrategicBarrier` is disjoint from `LegalBarrier`: In our established logic, a `StrategicBarrier` intersects with `lkif-action:Action` to reflect the deliberate, intentional, and exclusionary behavior of incumbents. A `LegalBarrier` intersects with `lkif-norm:Norm` to reflect government-imposed restrictions. Because LKIF-Core enforces disjointness between `Actions` and `Norms`, a barrier cannot simultaneously be a deliberate corporate action and a formal legislative rule.

[52] Noy and McGuinness, "Ontology Development 101: A Guide to Creating Your First Ontology", at 12.
[53] Ibid., at 13.
[54] Ibid., at 16.

Our model requires a distinction between transient events (the verbs in the previous) and endurants in order to take into account certain actors and transactions retaining their identity, yet having their properties (states) change in alternative scenarios where a law may be in effect.[55] A persistent issue in Ontology design is when to encode states or qualities as classes or properties. For example, the state or quality of being red can be encoded as a value of a color property (`Color="red"`) of a `Flower` class. Alternatively, we can reify "redness" as a subclass under `Color` (`Red` $\sqsubseteq$ `Color`), treating both as first-class conceptual entities in the class hierarchy. The recommended approach from the OWL Working Group is to adopt the latter approach through the "Value Partition" design pattern, where a quality space (e.g. `Health_Status`) is modeled as a class, and its values (e.g. `Good_Health`, `Poor_Health`) are represented as a collection of disjoint subclasses or as an enumerated list of individuals.[56]

While there are advantages and drawbacks to each approach, the goals and contemplated use cases for the reference ontology necessitate adopting the second, class-oriented approach. This choice is informed by the following considerations:

1. This is the method used by most foundational ontologies, including the ones that are used as basis for the core concepts of the ontology constructed for this work, such as LKIF. Adopting the same method can help with interoperability. (Note: In LKIF, subjective judgments or propositional attitudes are modeled as classes, i,e, as either a Qualification or Evaluative_Attitude. Thus, instead of simply applying an "is_prohibited" property to a particular action, the ontology creates a Norm class that normatively qualifies the situation by

[55] In ontology design, **endurants** are entities that, whenever they exist, are wholly present (i.e. whenever they are present, are present with all their parts). Endurants have essential properties (properties they must bear in all situations) and accidental properties (properties they bear in some possible situations)Because of the duality between essential and accidental properties, an endurant can qualitatively change in certain respects while maintaining their identity. Even if they can (or could have been) different in their accidental properties. Giancarlo Guizzardi, Nicola Guarino, and João Paulo A. Almeida. "Ontological Considerations About the Representation of Events and Endurants in Business Models". In: *Business Process Management*. Ed. by Marcello La Rosa, Peter Loos, and Oscar Pastor. Vol. 9850. Cham: Springer International Publishing, 2016, pp. 20–36. ISBN: 978-3-319-45347-7 978-3-319-45348-4. DOI: 10.1007/978-3-319-45348-4_2. URL: http://link.springer.com/10.1007/978-3-319-45348-4_2 (visited on 02/12/2026), at 2.

[56] Alan Rector. *Representing Specified Values in OWL: "Value Partitions" and "Value Sets"*. W3C Working Group. May 17, 2005. URL: https://www.w3.org/TR/2005/NOTE-swbp-specified-values-20050517/ (visited on 06/02/2026).

partitioning it into substates, e.g. "allowed" or "disallowed");

2. This approach converges with the Reified I/O logic methodology for encoding the normative content of the rules. As will be discussed later, on the latter mode, the norm itself is treated as an object (and thus corresponding to a class) that applies to or qualifies a certain case, partitioning the situation into sub states that are allowed or disallowed.

In the case of the reference ontology, the restrictive effects of legal provisions are modeled as classes: Instead of having a boolean property such as `restrictsCompetition:true`, it defines `CompetitionRestriction` as a standalone class. This allows us to build a specific hierarchy of restrictive states, e.g. `A1_GrantExclusiveRights`, corresponding with the constraint specified in A1, or `A4_RaiseCostOfEntryOrExit` for the state of affairs prohibited by A4. This will also enable the use of object properties to link restrictive states to to other entities. For example, we can use a property called `affectsSupplier` to connect `CompetitionRestriction` class to the specific `Supplier` that is subject to the market barrier.

5.2.5 STEP 5: Define the internal structure of classes

The internal structure of the classes can be defined through its properties or attributes.[57]. For every class in our ontology, we are concerned with both intrinsic and extrinsic properties:[58]

- **Intrinsic properties** - There are the essential, or inherent to the class itself. These properties are essential to the identity and nature of the class, independent of external factors or contexts. They are characteristics that an instance of the class possesses purely by being an instance of that class. For the class `Person`, intrinsic properties might include a `has_name` - since each legal person, whether an individual human being or a corporation, has a name.

- **Extrinsic properties** - These are context-dependent, relational attributes of a class. Extrinsic properties are those that depend on external factors or the context in which an instance of

[57] Also called slots in earlier documentation
[58] Noy and McGuinness, "Ontology Development 101: A Guide to Creating Your First Ontology", at 8.

the class exists. These properties are not essential to the identity of the class and can change depending on the environment, relationships, or interactions with other entities. For the class `Person`, extrinsic properties might include the person's current location, occupation, marital status, or the clothes they are wearing.[59]

Subclasses inherit the properties of their parent classes, and can have additional properties that are specific to them.[60] For example, the `Natural_Person` class can inherit the `has_name` property from the `Person` class, and can have additional properties such as `has_age` and `has_address`. The `Juridical_Entity` class can inherit the `has_name` property from the `Person` class, and can have additional properties such as `has_registration_number` and `has_legal_address`.

What follows is a description of the more important classes of the Reference Ontology, with insights to their internal structure in DL notation.

DominantSupplier: The internal structure of this class formally defines when an established firm holds market dominance.Its DL semantics can be encoded as:

$$DominantSupplier \equiv Supplier \sqcap \exists hasDominanceIndicator.HighMarketShare \sqcap \exists shieldedBy.BarrierToEntry \tag{5.1}$$

It is modeled as an equivalence class ($\equiv$) requiring the intersection ($\sqcap$) of three conditions. A firm is not considered a `DominantSupplier` solely by being a `Supplier` with a `HighMarketShare`. The logic mandates that it must simultaneously be `shieldedBy` a `BarrierToEntry`. This structure reflects the economic principle that a high market share does not reliably indicate market power

[59] A form of extrinsic properties that relate the class to other classes are mereological properties, i.e. a class can also have can have physical and abstract parts (e.g. the parts of an engine or the courses of a meal)

[60] Noy and McGuinness, "Ontology Development 101: A Guide to Creating Your First Ontology", at 9.

unless substantial entry barriers insulate the firm from competitive pressure and prevent rivals from entering the market.

Listing 5.5: Class Definition for Dominant Supplier

```
    ### Dominant Supplier Class ###

:DominantSupplier rdf:type owl:Class ;
    rdfs:subClassOf :Supplier ;
    rdfs:label "Dominant Supplier" ;
    rdfs:comment "A supplier with significant, durable market power,
       established by holding a high statutory market share while being
        shielded by substantial barriers to entry." ;
    owl:equivalentClass [
        rdf:type owl:Class ;
        owl:intersectionOf (
            :Supplier
            [ rdf:type owl:Restriction ;
              owl:onProperty :hasDominanceIndicator ;
              owl:someValuesFrom :HighMarketShare ]
            [ rdf:type owl:Restriction ;
              owl:onProperty :shieldedBy ;
              owl:someValuesFrom :BarrierToEntry ]
        )
    ] .

### Associated Object Properties ###

:hasDominanceIndicator rdf:type owl:ObjectProperty ;
```

```
    rdfs:domain :Supplier ;
    rdfs:range :DominanceIndicator ;
    rdfs:label "has dominance indicator" ;
    rdfs:comment "Links a supplier to specific evaluative indicators,
       such as a high market share or a lack of substitutability." .

:shieldedBy rdf:type owl:ObjectProperty ;
    rdfs:domain :DominantSupplier ;
    rdfs:range :BarrierToEntry ;
    rdfs:label "shielded by" ;
    rdfs:comment "Indicates the entry barriers that insulate the
       dominant supplier from potential rivals, preventing them from
       entering the market and undercutting it." .
```

CompetitionAssessment and CompetitionChecklist These classes model the procedural execution of evaluating regulations.

$$\begin{aligned} CompetitionAssessment \sqsubseteq lkif-action : ActionCompetitionChecklist \sqsubseteq \\ lkif-role : Epistemic\_RoleCompetitionChecklist \sqsubseteq \exists \\ screensFor.LimitsSuppliers \sqcap \exists \\ screensFor.LimitsAbilityToCompete \sqcap \exists \\ screensFor.ReducesIncentiveToCompete \sqcap \exists \\ screensFor.LimitsConsumerChoice \end{aligned} \tag{5.2}$$

The class `CompetitionAssessment` inherits from LKIF's `Action` class, as it represents an intentional change or evaluation brought about by an agent. The `CompetitionChecklist` inherits

from `Epistemic_Role`, which LKIF defines as an entity used specifically within a reasoning or inference process. The checklist's internal structure dictates that it must screen for four exact categories of anticompetitive effects: limiting the number/range of suppliers, limiting the ability of suppliers to compete, reducing the incentive to compete, and limiting choices/information available to customers.

Regulation and LegalBarrier This structure connects legislative documents to the market impediments they cause:

$$\begin{aligned} Regulation &\sqsubseteq lkif-norm : Regulation \\ LegalBarrier &\sqsubseteq BarrierToEntry \sqcap lkif-norm : Norm \\ Regulation \sqcap \exists imposesBarrier.LegalBarrier &\sqsubseteq \\ lkif-norm : Regulation\exists & \\ imposesBarrier.(BarrierToEntry \sqcap lkif-norm : Norm) & \end{aligned} \tag{5.3}$$

A `Regulation` is grounded in LKIF as a legal document bearing one or more norms uttered by a legislative body. A `LegalBarrier` is structured as a dual-inheritance class: it is simultaneously an economic `BarrierToEntry` and a legal `lkif-norm:Norm`. If a regulation imposes a legal barrier, the ontology formally recognizes it as a normative constraint that alters market dynamics by legally permitting or prohibiting behavior.

LimitsSuppliers This class unpacks the system of constraints screened by Section A of the OECD threshold test checklist, e.g. the specific mechanisms by which a regulation might restrict market entry.

$$\begin{aligned} LimitsSuppliers &\sqsubseteq ChecklistCategory \\ GrantsExclusiveRights &\sqsubseteq LimitsSuppliers \\ RequiresLicenseOrPermit &\sqsubseteq LimitsSuppliers \\ RaisesEntryOrExitCost &\sqsubseteq LimitsSuppliers \end{aligned} \tag{5.4}$$

As a subclass of `ChecklistCategory`, `LimitsSuppliers` acts as a parent class for specific regulatory provisions that reduce rivalry. The internal subclasses correspond directly to the OECD toolkit's questions, screening for provisions that grant exclusive rights (A1), establish authorization or licensing processes (A2), limit the ability of certain suppliers to provide goods (A3), or significantly raise the costs of entry or exit (A4).

StrategicBarrier This class captures barriers generated by incumbent behavior rather than exogenous market conditions or laws.

$$StrategicBarrier \sqsubseteq BarrierToEntry \sqcap lkif - action : Action \tag{5.5}$$

Like `LegalBarrier`, `StrategicBarrier` uses multiple inheritance. It is defined as a `BarrierToEntry` intersected with LKIF's `Action` class. This intersection explicitly models the fact that strategic barriers—such as exclusive dealing or raising rivals' costs—are not natural market conditions, but rather the result of intentional, deliberate, and tactical exclusionary acts performed by incumbent agents seeking to deter potential entrants.

5.2.6 STEP 6: Define the attribute restrictions

The properties of a class can have restrictions that define the cardinality of the property, the value of the property, or the relationship between the property and other properties.[61] We can define the cardinality of an attribute - how many values a property can have. The `has_name` for a person can have a single cardinality - that is, a person is allowed to only have one legal name. Other properties can have multiple cardinality. For example, the `has_child` property of a `Natural_Person` class can have a restriction that specifies that a child can have at most two parents, or several friends. We can also define restrictions for acceptable values that can be entered for each property: The `has_name` property can have a restriction that specifies that the value of the property must be a string(i.e. a series of text characters), or that the `has_age` property can have a restriction that specifies that the value of the property must be a positive integer. By specifying the domain and **range** of an attribute, we can place restrictions on the relationships of classes. The **domain** of a property refers to the set of all objects that can have that property asserted about it.[62] The **range** of a property, on the other hand, the set of all objects that can be the value of the property.[63] For example, the fact that a `Law` can contain many `Rules` can be modelled by the `has_rule` attribute. The `has_child` property can also have a restriction that specifies that a child must be a `Natural_Person` instance.

When defining a domain or range of an attribute, Noy(2001) recommends finding the most general classes or class that can serve the purpose. Nevertheless the domain or the range should not be too general, i.e. the classes in the domain of an attribute should be described by the attribute, and the instances of all the classes in the range of an attribute should be potential values for the attribute.[64]

The ontology establishes relational boundaries using domain and range restrictions, ensuring that properties only link appropriate classes. The property restrictions can be organized along the following modules:

[61] Noy and McGuinness, "Ontology Development 101: A Guide to Creating Your First Ontology", at 9.
[62] De Bellis, *A Practical Guide to Building OWL Ontologies*, at 26.
[63] Ibid.
[64] Noy and McGuinness, "Ontology Development 101: A Guide to Creating Your First Ontology", at 10.

Assessment Framework This module grounds the evaluation process in formal legal semantics, ensuring that the assessment of a policy is treated as a deliberate action evaluating a legal norm. - `CompetitionAssessment` represents the deliberate process of evaluating laws or policies. It is a sub-class of `lkif-action:Action`, defined as an intentional change brought about by a single agent; `Regulation` represents a law or policy being evaluated. It is grounded in `lkif-norm:Regulation`, which bears one or more norms uttered by a legislative body; `CompetitionChecklist` models a screening tool used in a reasoning process, making it a sub-class of `lkif-role:Epistemic_Role`.

$$
\begin{aligned}
Regulation &\sqsubseteq lkif - norm : Regulation \\
CompetitionAssessment &\sqsubseteq lkif - action : Action \\
CompetitionChecklist &\sqsubseteq lkif - role : Epistemic_{R}ole \sqcap \exists \\
&screensFor.LimitsSuppliers \sqcap \exists \\
&screensFor.LimitsAbilityToCompete \sqcap \exists \\
&screensFor.ReducesIncentiveToCompete \sqcap \exists \\
&screensFor.LimitsConsumerChoice
\end{aligned}
\tag{5.6}
$$

Competition Checklist Modules The evaluation of Category A relies on object properties that link the legal assessment to the specific regulatory provisions and the economic barriers they generate: The `screensFor` property links the overall `CompetitionChecklist` to the specific `LimitsSuppliers` categories. It forces the assessment to explicitly check for the presence of the five criteria in Section A; `evaluatesRegulation` links the `CompetitionAssessment` to the `Regulation` being evaluated. In DL constraints, the inverse of this property ($evaluatesRegulation^{-1}$) is used to map the findings of the checklist back to the regulation itself; the `imposesBarrier` property links a `Regulation` to the specific `BarrierToEntry` it creates.

$$
\begin{aligned}
LimitsSuppliers &\sqsubseteq ChecklistCategory \\
GrantsExclusiveRights &\sqsubseteq LimitsSuppliers \\
RequiresLicenseOrPermit &\sqsubseteq LimitsSuppliers \\
LimitsSupplierParticipation &\sqsubseteq LimitsSuppliers \\
RaisesEntryOrExitCost &\sqsubseteq LimitsSuppliers \\
CreatesGeographicalBarrier &\sqsubseteq LimitsSuppliers
\end{aligned}
\tag{5.7}
$$

Modeling Entry Barriers The ontology uses complex constraints to automatically infer the type of entry barrier generated by a regulation based on which Category A sub-class it triggers:

- **Legal/Normative Constraints:** If the checklist determines that a regulation grants exclusive rights (A1) or establishes a license, permit, or authorisation process (A2), the ontology dictates that the regulation inherently `imposesBarrier` of the type `LegalBarrier`. Because it acts as a formal legal limitation, this barrier is simultaneously classed as an `lkif-norm:Norm`.
- **Structural Constraints:** If the checklist determines that a regulation significantly raises the cost of entry or exit (A4), the ontology infers that the regulation `imposesBarrier` of the type `:StructuralBarrier` (such as sunk costs).

A regulation evaluated as containing A1 or A2 provisions imposes a Legal Barrier, defined formally as $Regulation \sqcap \exists evaluatesRegulation^{-1}.(\exists screensFor.(GrantsExclusiveRights \sqcup RequiresLicenseOrPermit)) \sqsubseteq \exists imposesBarrier.LegalBarrier.$

On the other hand,a regulation evaluated as containing an A4 provision imposes a Structural Barrier - $Regulation \sqcap \exists evaluatesRegulation^{-1}.(\exists screensFor.RaisesEntryOrExitCost) \sqsubseteq \exists imposesBarrier.StructuralBarrier$

Classification of a Dominant Supplier In our custom ontology, based on your previous instruction to modify the determination to be based on market share, the logic dictating whether a $Supplier$ is classified as a $DominantSupplier$ is formalized using the following Description Logic (DL) equivalence axiom:

$$DominantSupplier \equiv Supplier \sqcap \exists hasDominanceIndicator.HighMarketShare \sqcap \exists shieldedBy.BarrierToEntry \tag{5.8}$$

This logical equivalence dictates that a firm cannot be deemed dominant based on a single isolated metric. Instead, it must simultaneously satisfy three intersecting ($\sqcap$) conditions:

- It must be a $Supplier$: The entity must actively operate as a participant within the market.
- It must possess a $HighMarketShare$: It must meet a statutory threshold for market share. Competition authorities frequently use market share thresholds as a starting point to provide clarity on situations in which firms will or will not be considered dominant. For example, the European Commission indicates that a firm with a market share of less than 40% is unlikely to be dominant, while US agencies suggest that dominant firms generally have a market share of at least 50%.
- It must be $shieldedBy$ a $BarrierToEntry$: The ontology's logic strictly requires the presence of entry barriers because market shares alone are not a reliable indicator of dominance or market power; by themselves, they fail to capture dynamic competitive pressures from substitution. A firm with a high market share is only considered to have a durable dominant position if it is insulated from competitive pressure by entry barriers that prevent potential rivals from entering the market and undercutting it.

By intersecting these requirements, the ontology logically ensures that a high market share only triggers a $DominantSupplier$ classification when it is actively protected by structural, legal, or strategic market impediments.

Modelling Section A Constraints In the custom LKIF-aligned ontology, Section A of the OECD Competition Assessment Checklist, which evaluates whether a regulation "Limits the number or range of suppliers", is formally modeled as the `:LimitsSuppliers` class. Because this category serves as a diagnostic screening tool during an assessment, it inherits from `lkif-role:Epistemic_Role`, which designates entities used specifically within reasoning or inference processes.

To capture the specific mechanisms by which a regulation might restrict market participation, `:LimitsSuppliers` is broken down into five distinct sub-classes that correspond directly to the OECD toolkit's checklist questions:

- `:GrantsExclusiveRights` (A1): Screens for provisions that grant exclusive rights for a supplier to provide goods or services.
- `:RequiresLicenseOrPermit` (A2): Screens for provisions that establish a license, permit or authorisation process as a requirement of operation.
- `:LimitsSupplierParticipation` (A3): Screens for provisions that limit the ability of some suppliers to provide a good or service.
- `:RaisesEntryOrExitCost` (A4): Screens for provisions that significantly raise cost of entry or exit by a supplier.
- `:CreatesGeographicalBarrier` (A5): Screens for provisions that **create a geographical barrier for companies to supply goods, services or labour, or invest capital.

The evaluation of Section A relies on specific object properties to link the legal assessment to the regulatory provisions and the economic barriers they generate:

- `screensFor`: This property links the `:CompetitionChecklist` to the `:LimitsSuppliers` sub-categories, structurally mandating that the assessment checks for the presence of the five A-level provisions.

- `imposesBarrier`: This property maps the ':Regulation' being evaluated to the specific ':BarrierToEntry' it creates in the market.

The ontology uses complex DL constraints to automatically infer the type of entry barrier generated by a regulation based on which Section A sub-class it triggers.

Imposing Legal Barriers: If the checklist determines that a regulation grants exclusive rights (A1) or establishes a license or permit process (A2), the ontology dictates that the regulation inherently imposes a `:LegalBarrier`. Because these act as formal legal limitations, they are simultaneously classed under `lkif-norm:Norm`.

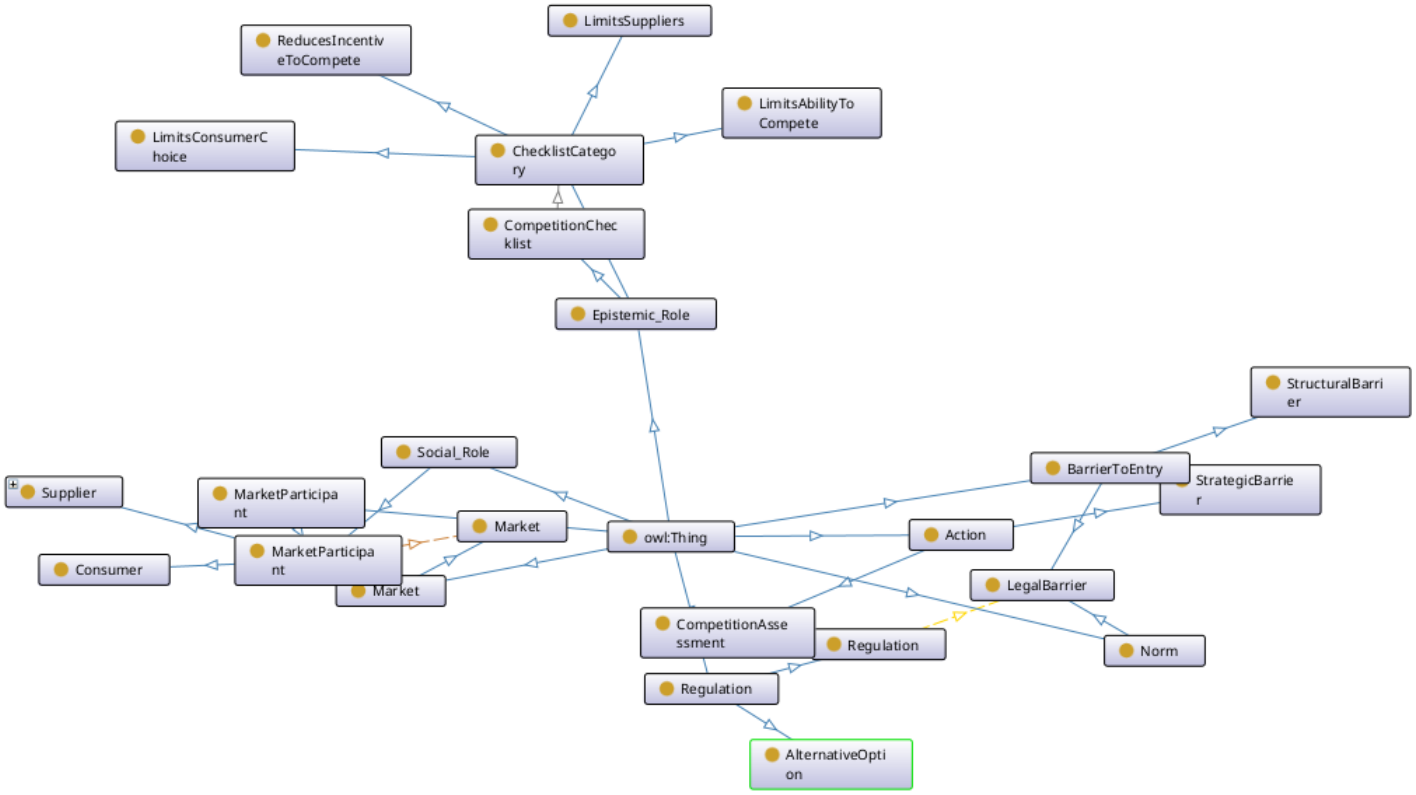
$$Regulation \sqcap \exists evaluatesRegulation^{-1}.(\exists screensFor.(GrantsExclusiveRights \sqcup RequiresLicenseOrPermit)) \sqsubseteq \exists imposesBarrier.LegalBarrier \tag{5.9}$$

Imposing Structural Barriers: If the checklist determines that a regulation significantly raises the cost of entry or exit (A4), the ontology infers that the regulation inherently contributes to a `:StructuralBarrier` (such as large sunk costs).

$$Regulation \sqcap \exists evaluatesRegulation^{-1}.(\exists screensFor.RaisesEntryOrExitCost) \sqsubseteq \exists imposesBarrier.StructuralBarrier \tag{5.10}$$

By structuring Section A this way, the ontology seamlessly bridges the OECD's procedural screening questions with the formal economic and legal market constraints they generate.

Figure 5.3: Initial class diagram



Chapter 6

Encoding the Normative Component of the Domain

Ontologies only give us a part of the picture. Besides the entities, their attributes and interactions to be modeled in the OECD Guidelines - all these are subject to constraints and transformations based on law. These only provide static data about the semantics of entities and their interactions - but these do not reflect the legal constraints that act upon those objects, and how the semantics could be qualified, transformed, or annulled by such constraints.

In the previous chapters, we have provided an overview of the semantics of the OECD competition assessment checklist (particularly section A), and how these can be encoded as TBox information - with both a logical account (in Description Logic) and a programmatic account (in OWL). In this chapter, we will provide an overview of how the ABox of the domain can be represented through a logical account (through Reified I/O logic) and a programmatic account (through SHACL).

The ontology alone cannot carry the weight of automated legal reasoning. Description Logic (as encoded in OWL) only covers definition and description of concepts and relations. While this allows for basic reasoning - it does not provide sufficient resolution to represent obligations and

permissions, to support applications (such as prediction and compliance checking).[1] The OWL ontology does not take into account the defeasibility of legal knowledge:

1. That provisions may be subject to different, even conflicting interpretations;
2. Additional definitions and rules may need to be provided for specific contexts;
3. New rules may supplant existing ones.

In previous chapters, we have laid out the foundation for the encoding of the law: Terminological information, which can be read into a logical account (through Description Logic) and computationally encoded into an OWL ontology. The normative content of the law, on the other hand, has a logical account that via Reified I/O Logic. This section examines the computational encoding of the norms, through the Shapes Constraint Language (SHACL).

6.1 Representation of Norms

6.1.1 Availability of Multiple Logical Systems

Consider moving to an earlier chapter

In stating that we will translate legal rules into a logical encoding, we mean "logic" as a formal method that can support deductive reasoning. That is, proving a conclusion by means of at least two other propositions.[2] The term includes not just Aristotelian syllogism, but can accomodate other forms of deductive inferences, such as the logic of alternatives, compound propositions, and of relationships, and the study of propositions themselves.[3] Burgin (2022) provides a high-level overview of the evolution of logical systems: From loose collection of rules related to belief systems

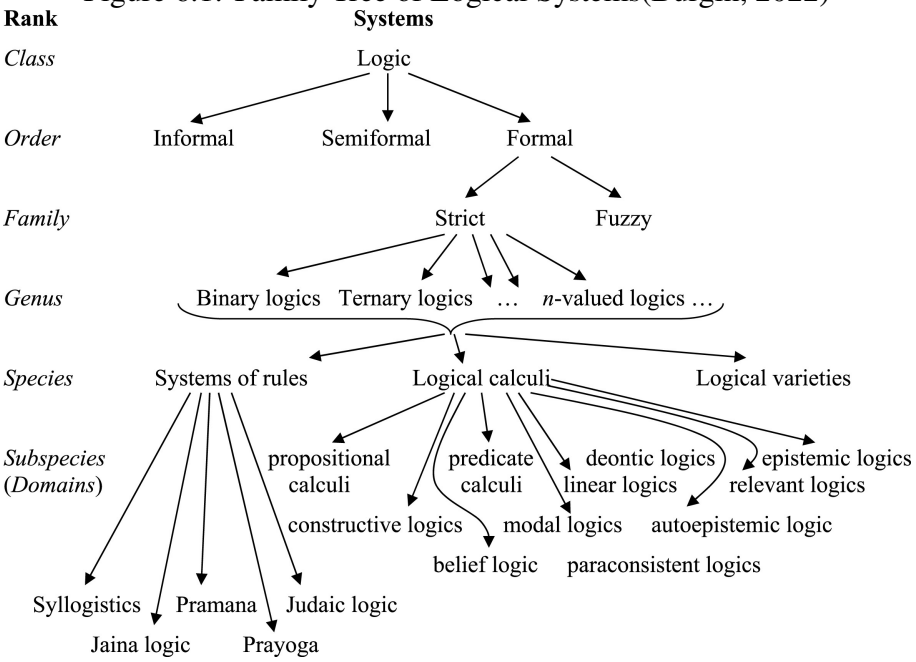
[1] Livio Robaldo et al. "Formalizing GDPR Provisions in Reified I/O Logic: The DAPRECO Knowledge Base". In: *Journal of Logic, Language and Information* 29.4 (Dec. 2020), pp. 401–449. ISSN: 0925-8531, 1572-9583. DOI: 10.1007/s10849-019-09309-z. URL: http://link.springer.com/10.1007/s10849-019-09309-z (visited on 11/25/2024), at 5-6.

[2] Aldisert, Clowney, and Peterson, "Logic for Law Students: How to Think Like a Lawyer", at 2.

[3] Guest, "Logic in the Law".

to more modern, formalized logics.[4] The diversity of logics can provide tools for the representation of various aspects of knowledge. Each logic can capture and emphasize a certain level of description, or comprehend specific problems.

Figure 6.1: Family Tree of Logical Systems(Burgin, 2022)



The development of novel logical systems now allow us to have a more focused view of a problem.[5] For example: Triadic logics which allows for intermediate truth values, rejecting the law of the excluded middle; A fuzzy logic, which has instead of True or False, an infinite continuum of possible values, along with a more informal process of inference. Then there are deontic logics, with new operations such as “obligatory", “permitted", and “forbidden".[6]

Explain these logics. Why do we need to enumerate them

How do these logics relate to each other, and how it leads to the logic we will actually

[4] See generally Mark Burgin. “Evolution of logic as an information processing mechanism in advanced biological systems”. In: *Bio Systems* 221 (2022), p. 104758. ISSN: 0303-2647. DOI: 10.1016/j.biosystems.2022.104758. URL: https://www.sciencedirect.com/science/article/pii/S0303264722001393.

[5] Haack, “On Logic in the Law”, at.

[6] Deontic logics in particular have attracted legal theorists as a way to make formal, rigorous representations of the structure of legal orders. A variation of the idea of deontic modes is a component of the encoding sytem proposed later in this chapter ibid., at 11-12.

6.1.2 Legal Norms as Conditional Inferences

Legal provisions may be restated into atomic inference rules, which have the structure - If P then Q. It is also possible to state a rule categorically as simply Q, but this should be rare in operation.[7]

In more conventional forms of logic, we can readily represent factual statements. Take for example the proposition, earlier made explicit as a fact in the ontology, that corporations are also persons:

$$\text{All Corporations are Persons}(\text{All } S \text{ is } P) \tag{6.1}$$

The predicate Persons is descriptive of the subject Corporations. If the proposition is admitted, it follows that no Corporations are not Persons. If the proposition is denied, then it follows that some Corporations are not Persons.

However, let us take a normative or legal statement, “Any person who shall abuse its market dominance shall be guilty of a criminal offense". The predicate “guilty of a criminal offense" is not necessarily descriptive of the subject "person who abuses market dominance". The relationship between the components of the proposition hinges on the injunctive "shall", which is not descriptive of what is, but instead denotes **what ought to be** under certain contingencies.[8] Normative propositions are more comparable to the structure of causal inferences: If p then q. Instead of making factual predictions, however, they are statements of what ought to be.[9] Sartor states that legal provisions

[7] Sartor formally defines an inference rule as a logical formula of the type $\langle r : p_0 \, if \, p_1 \, and \ldots and \, p_n \rangle$ where r is the rule name, each p_i is a literal, *if* is the connective relating antecedent and consequent in inference rules; *and* expresses logical conjunction. Categorical inference rules simply have simply the form $\langle r : p \rangle$ where p is a literal. Giovanni Sartor. “A Formal Model of Legal Argumentation”. In: *Ratio Juris* 7.2 (July 1994), pp. 177–211. ISSN: 0952-1917, 1467-9337. DOI: 10.1111/j.1467-9337.1994.tb00175.x. URL: https://onlinelibrary.wiley.com/doi/10.1111/j.1467-9337.1994.tb00175.x (visited on 06/20/2023), at 179.

[8] Guest, “Logic in the Law”, at 183-184.

[9] Guest also clarifies that these are not necessarily imperative statements or commands. ibid., at 184.

may be restated into such atomic inference rules, with the basic if p then q structure.[10] It may also be possible to state a rule categorically as simply q, but this should be rare in operation.[11]

Take for example rule A1 in the OECD Guidelines, that a provision should be flagged if it "Grants exclusive rights for a supplier to provide goods and services". This can be broken down to several inference rules:

Exclusive supplier not Single Supplier

If [Law Requires Single Supplier] then [Rights are Exclusive] (6.2)

If [Rights are Exclusive] then [Flagged] (6.3)

Where [Rights are Exclusive] stands for p and [Flagged] stands for q.

What is an exclusive right? How about alternatively, a permit system.

Initially, we can think of rules of substantive law as statements of the specific factual conditions upon which specific consequences depend. The applicability of the condition can be contingent on several conditions, as well as the absence of exceptions.[12] Thus:

[10] The consequent of each rule is a litera;. and the antecedent is a conjunction of literals. A literal is an atomic formula or the negation thereof. A positive literal has the form 'p(x)' where 'p' is a predicate symbol and 'x' is a list of terms. On the other hand, a negative literal has the form 'not p(x)' where 'not' is a logical negation. The complement $\bar{q}$ of a literal q denotes the literal opposed to q : If q is a positive literal p, then $\bar{q}$ represents the negative literal not p ; if q is the negative literal not p, then $\bar{q}$ represents the positive literal p. Sartor, "A Formal Model of Legal Argumentation", at 179.

[11] Sartor's formalization also admits of these so-called "degenerate inference rules". These enable unconditional derivation of any instance of their conclusion A. These categorical inference rules can be used to express some forms of ungrounded assertion, such as:1. Statements of undisputed empirical evidence (the facts that justify a law or court decision); 2. Basic (and very general) normative postulates; 3. Tentatively advanced propositions for which no ground is currently available ibid., at 179.

[12] Jerome Michael and Mortimer J. Adler. "The Trial of an Issue of Fact: I". in: *Columbia Law Review* 34.7 (Nov. 1934), pp. 1224–1306. ISSN: 00101958. DOI: 10.2307/1116103. JSTOR: 1116103. URL: https://www.jstor.org/stable/1116103?origin=crossref (visited on 12/11/2024), at 1241.

If events $x_1 \ldots x_n$ is the case, and unless there are $y_1 \ldots y_n$, then z is the case. (6.4)

A legal system can be represented as a body of such propositions that can be evaluated not based on truth or falsity, but by some other normative standard (such as social benefit, or compliance with other higher rules). Once we have formal representations, the next step would be to apply analytical methods grounded in logic. We can trace the chain of inferences (e.g. via *modus ponens*), discover other rules, even look for potential inconsistencies.[13]

Once we have formal representations, the next step would be to apply analytical methods grounded in logic. We can trace the chain of inferences (via *modus ponens*), discover other rules, even look for potential inconsistencies.

6.1.3 Deontic Logic

Legal statements are for the most part, not composed of factual statements. They do not describe the state of the world as it is, but how it ought to be. They cannot be assessed for truth values. Furthermore, legal conclusions are arrived at under an informational environment marked by incompleteness, uncertainty, and inconsistency.[14] Logicians have since developed a form of logic, called Deontic Logic, which is not concerned with True or False, but **oughtness**.[15]Instead of these binary values, Deontic Logic accommodates six normative states:

[13]Inference rules are mono-directional, to be used/understood only forward (*modo ponente*) and not backward (*modo tollente*). The consequent q can be derived whenever the antecedent p is satisfied. However, the negation of p cannot be derived when q is assumed to be false. The "if" connective in inference rules is not the same "if" in logical conditionals. Sartor, "A Formal Model of Legal Argumentation", at 179.

[14]See Kathleen Freeman and Arthur M. Farley. "A Model of Argumentation and Its Application to Legal Reasoning". In: *Artificial Intelligence and Law* 4.3-4 (1996), pp. 163–197, at 165.

[15]See G.H. von Wright. "Deontic Logic". In: *Mind* 60.237 (Jan. 1951), pp. 1–15, for the original use of the term and the first modern systemization of the field; See also Paul McNamara and Frederik Van De Putte. "Deontic Logic". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Spring 2022. Metaphysics Research Lab, Stanford University, 2022. URL: https://plato.stanford.edu/archives/spr2022/entries/logic-deontic/ (visited on 06/08/2022), for an updated overview.

1. It is obligatory that (OB)

2. It is permissible that (PE)

3. It is impermissible that (IM)

4. It is omissible that (OM)

5. It is optional that (OP)

6. It is is non-optional that (NO)

Deontic Logic was influenced by modal logic (which concerns modalities, or expressions that qualify the truth of propositions, i.e., necessity and probability) Although notions of Deontic Logic have been explored in fourteenth century Europe as well as Islamic thought (in the 10th century), its modern version grounded in symbolic logic is based on the work of von Wright (1951).[16]. Under von Wright(1951)'s classic formulation, it is concerned with the following modes of obligation:[17]

- **Obligatory** - That which we ought to do
- **Permitted** - That which we are allowed to do
- **Forbidden** - That which we must not do

For von Wright the starting point of his deontic system is the concept of "Permitted" as the basic operator - e.g. a proposition can ϕ is Permitted, $P\phi$. Other operators can then be defined in terms of P:[18]

[16] McNamara and Van De Putte, "Deontic Logic", at § 1.

[17] Von Wright, "Deontic Logic", at 1.

[18] Donald Nute, ed. *Defeasible Deontic Logic*. Dordrecht: Springer Netherlands, 1997. ISBN: 978-90-481-4874-5 978-94-015-8851-5. DOI: 10.1007/978-94-015-8851-5. URL: http://link.springer.com/10.1007/978-94-015-8851-5 (visited on 11/23/2024), at 2.

$$F\phi =_{df} \neg P\phi \text{ - something Forbidden is not Permitted} \quad (6.5)$$

$$O\phi =_{df} \neg P\neg\phi \text{ - something Obligatory is something not Permitted not to do} \quad (6.6)$$

Deontic logic was later axiomatized and developed to what is now known as Standard Deontic Logic (SDL). Under SDL, the primary operator is Obligation, denoted as by the symbol $\bigcirc$ (or "ought"). The Permitted operator can be defined as:[19]

$$PE\phi =_{df} \neg \bigcirc \neg\phi \quad (6.7)$$

That is, ϕ is Permitted if and only if it is not am Obligation that not ϕ. We can thus construct all the other operators in terms of $\bigcirc$ (See Table 6.1 at 205 below).[20]

The content of this rule - Supplier is not allowed to sell

Recasting the earlier example under the Deontic mode:

If [Law Requires Single Supplier] then [Rights are Exclusive] (6.8)

If [Rights are Exclusive] then $\bigcirc$ [Flagged] (6.9)

The inference that the law should be flagged is neither True nor False, but has the deontic modality of Obligation. Once represented formally, it may be possible to evaluate a specific statement based

[19] Nute, *Defeasible Deontic Logic*, at 2.

[20] McNamara and Van De Putte, "Deontic Logic", at § 1.2.

Table 6.1: SDL Definitions of Deontic Operators

Definition	**Implication**	**Example**
$\bigcirc$(OB)	A proposition is obligatory if it must occur	It is OBligatory to pay taxes
PE $\phi =_{df} \neg \bigcirc \neg \phi$	A proposition is permissible iff (if and only if) its negation is not obligatory	It is PErmitted to drive a car
IM $\phi =_{df} \bigcirc \neg \phi$	A proposition is impermissible iff (if and only if) its negation is obligatory	It is IMpermissible to smoke in a restaurant
OM $\phi =_{df} \neg \bigcirc \phi$	A proposition is omissible iff it is not obligatory (can be omitted or not done without violating a norm)	It is OMissible to attend that party (you can attend or not attend)
OP $\phi =_{df} (\neg \bigcirc \phi \wedge \neg \bigcirc \neg \phi)$	A proposition is optional iff neither it nor its negation is obligatory	It is OPtional to work from home
NO $\phi =_{df} (\bigcirc \phi \vee \bigcirc \neg \phi)$	A proposition is non-optional iff it is either obligatory or impermissible	It is NOn-optional to wear a seatbelt while driving

on the axioms and theorems of the chosen system of deontic logic. For example, in von Wright's classical system, there exists the Principle of Deontic Distribution which provides that: "If an act is the disjunction ("or") of two other acts, then the proposition that the disjunction is permitted is equivalent to the disjunction of the propositions that the first act is permitted and the proposition that the second act is permitted":[21]

$$P(\phi \vee \psi) \leftrightarrow P\phi \vee P\psi \tag{6.10}$$

Applying this to the proposition that a Supplier is permitted to supply goods or services, then the permission is distributed individually to the supply of goods as well as the supply of services. While

[21] Nute, *Defeasible Deontic Logic*, at 2.

this distributive property is a feature of this particular system of deontic logic, it is not universally accepted, and we can discard this axiom if it conflicts with our normative intuition.

Standard Deontic Logic may be insufficient for modelling competition norms, as it lacks the following attributes:

1. **Granularity** - Legal rules quantify over specific actions, e.g. applying dissimilar conditions to equivalent transactions. SDL is not sufficient to capture the internal structure of these structures;

2. **Jorgensen's Dilemma** - Norms are prescriptions, not descriptions. They are neither true nor false. Standard logic is concerned with truth values, and can create paradoxes when applied to norms;

3. **Defeasibility** - Competition law findings are presumptive. E.g. a market share of above 50 percent presumes dominance, but this can be rebutted. SDL is typically monotonic (once conclusion is derived, it cannot be retracted).

6.1.4 Defeasible Deontic Logic

Another attribute of legal propositions is that they are **defeasible**. This means that they are tentative - accepted until some other proposition - a new fact that activates an exception, better evidence, or even a higher law - defeats our original proposition.[22] Most of our practices are undergirded by propositions that are not deductively valid (i.e. they are not deductions of observable phenomena derived from definition that hold universally without exception) Instead, we are reliant on common sense generalizations that hold only for the most part.[23] In real life, conclusions are

[22] See generally Giovanni Sartor. "Defeasibility in Legal Reasoning". In: *Rechtstheorie* 24.3 (1993), pp. 281–316.

[23] HLA Hart notes the centrality of defeasible reasoning in the law (especially in the common law tradition). In a judicial setting, a judge must make a determination based on an incomplete facts i.e. whatever the parties present, and whatever the rules of admissibility determines as germane. Robert Koons. "Defeasible Reasoning". In: *The Stanford Encyclopedia of Philosophy*. Ed. by Edward N. Zalta. Summer 2022. Metaphysics Research Lab, Stanford University, 2022. URL: https://plato.stanford.edu/archives/sum2022/entries/reasoning-defeasible/ (visited on 12/29/2023), at Sec. 1.1, par. 1-3.

arrived at tentatively, based on partial information. That is, with the possibility of retraction when more complete information is learned.

Unlike formalized deductive logic, defeasible reasoning is **non-monotonic**.[24] That is, it allows new information to invalidate previous conclusions.[25]

Substantive legal provisions often have a **positive condition**, the event or circumstance that must obtain for the purported legal consequence to be arrived at. At the same time, these conditions are most likely subject to exceptions - elements that according to some antecedent norms has to be absent in order for the legal consequence fo apply: The sum of positive conditions embody the determination of the legislator of what circumstances should normally give rise to the legal consequences. On the other hand, the exceptions represent special circumstances that can override the positive conditions, making the legal consequences not applicable. Legal conclusions are often subordinated structures: The presence of other legal provisions (that are of equal or higher priority in a hierarchy of norms), which may provide (or negate) conditions and exceptions.[26] The goal of legal reasoning in actual cases is to show that certain acts, claims, decisions comply or does not comply with the law. This requires demonstrating that the presence (or absence) of conditions and exceptions.[27] Furthermore, the conclusions that can be derived from legal norms are based not on truth but on justifiable belief - whatever can be shown to be the case. And it is considered reasonable to hold on to those justified beliefs until additional evidence compels us to change our

[24]Monotony is the property of some forms of logic; through which the set of conclusions following from a given body of knowledge itself, i.e., once a given conclusion has been reached , it cannot be undone by the addition of further information.Monotony is desirable if one is building a logical system applicable to rigorous mathematical reasoning. For these systems, we need conclusions to always necessarily follow from premises, and this relationship (between premises and conclusions) do not and cannot become void through the provision of additional facts. G. Aldo Antonelli. *Grounded Consequence for Defeasible Logic*. New York: Cambridge University Press, 2005, at x-xi.

[25]Ibid., at xi.

[26]This arises from what Stuart Hampshire calls the "inexhaustability of description" Any situation can embody an inexhaustible set of features, but we can only confront and understand part of it at any given time. See Juan Carlos Bayon. “Why Is Legal Reasoning Defeasible?” In: *Diritto & Questioni Pubbliche* 2 (2002), pp. 1–18, at 3; Citing Stuart Hampshire, ed. *Public and Private Morality*. Cambridge: Cambridge University Press, 1991. 143 pp. ISBN: 978-0-521-22084-2 978-0-521-29352-5, at 30.

[27]Bayon, “Why Is Legal Reasoning Defeasible?”, at 3.

minds.[28] Finally, legal conclusions are arrived at based on knowledge that is incomplete, uncertain, and inconsistent[29] - on plausibility rather than truth. Despite this, an adequate formalization of defeasible reasoning should provide a sound basis of what to believe.

Various such formalizations have been developed to embody defeasibility of reasoning. For our purposes, a system of defeasible reasoning should allow for the representation of various propositions and their attributes: 1. Atomic "facts" that are taken as a given; 2. Rules (whether or not they are subject to exceptions); 3. Defeating propositions and/or superiority relationships; 4. In the case of legal statements especially, their deontic modal values (Obligatory, Permitted, or Forbidden). A system of defeasible reasoning should also enable operations on these propositions, such as resolving conflicts and making plausible inferences. For example - through prioritization of certain rules and/or the evaluation of supporting or undercutting evidence.[30]

In Defeasible Deontic Logic (DDL), legal norms are the positive conditions that prescribe behavior through Permission, Obligation, and Prohibitions. These norms may be subject to exceptions (which are also expressed as norms).[31] DDL allows for the representation of facts, defined as whatever can be considered as conclusive unambiguous statements. Facts can include: either a state of affairs or actions already performed (both considered to always hold true). Based on our ontological definitions, we can state that "Acme Inc. is a corporation" through:

[28] Bayon, "Why Is Legal Reasoning Defeasible?", at 6.

[29] See Freeman and Farley, "A Model of Argumentation and Its Application to Legal Reasoning", at 165.

[30] Hanif Bhuiyan et al. "Traffic Rules Encoding Using Defeasible Deontic Logic". In: *Frontiers in Artificial Intelligence and Applications*. Ed. by Serena Villata, Jakub Harašta, and Petr Křemen. IOS Press, Dec. 1, 2020. ISBN: 978-1-64368-150-4 978-1-64368-151-1. DOI: 10.3233/FAIA200844. URL: http://ebooks.iospress.nl/doi/10.3233/FAIA200844 (visited on 11/23/2024), at 9; See also Sanjay Modgil and Henry Prakken. "The ASPIC+framework for Structured Argumentation: A Tutorial". In: *Argument & Computation* 5.1 (Jan. 2, 2014), pp. 31–62. ISSN: 1946-2166, 1946-2174. DOI: 10.1080/19462166.2013.869766. URL: http://content.iospress.com/doi/10.1080/19462166.2013.869766 (visited on 11/27/2023).

[31] Hanif Bhuiyan et al. "A Methodology for Encoding Regulatory Rules". In: *Proceedings of the 4th International Workshop on MIning and REasoning with Legal Texts Co-Located with the 32nd International Conference on Legal Knowledge and Information Systems (JURIX 2019)*. International Workshop on MIning and REasoning with Legal Texts 2019. Vol. 2632. Madrid, Spain: Rheinisch-Westfaelische Technische Hochschule Aachen, Dec. 11, 2019, at 2.

$$\text{Corporation(Acme Inc.)} \tag{6.11}$$

A rule in DDL is a relationship between a set of antecedents or premises (clauses), represented as $X_1, ..., X_n$ and the consequent conclusion or conclusion (effect) of the rule, is represented as Y. The strength of the relationship between the premises and conclusion allows us to differentiate between strict rules, defeasible rules, and defeaters:[32]

Strict rules (encoded as $X_1, ..., X_n \rightarrow Y$) are inferences in the classical propositional sense. If the premise is indisputable, then so is the conclusion. E.g. "A Corporation is a Supplier":

$$\text{Corporation(Acme Inc.)} \rightarrow \text{Supplier(Acme Inc.)} \tag{6.12}$$

Defeasible rules (encoded as $X_1, ..., X_n \Rightarrow Y$) are inferences that are generally true, but can be defeated by other information. An example in the guidelines is that a Supplier cannot be an exclusive provider unless the economic sector allows for a natural monopoly:

Sector is mentioned. How do we define a sector. May need to cite the OECD

$$\text{Corporation(Acme Inc.)} \Rightarrow \text{ExclusiveSupplier(Acme Inc.)} \tag{6.13}$$

From this, we can conclude that a corporation can be an exclusive supplier, unless there is evidence to the contrary.

Respond to this comment - We are not looking at the behavior, but what the law says. What the text of the law indicates. How it relates to other texts.

[32] Ibid., at 8-9.

Defeaters (encoded as $X_1, ..., X_n \rightsquigarrow Y$) are rules that can prevent a conclusion. Building on the previous example, we can maintain that:

$$\neg(\text{Sector\_AllowsNaturalMonopoly}(\text{Acme Inc.})) \rightsquigarrow \neg\text{ExclusiveSupplier}(\text{Acme Inc.}) \tag{6.14}$$

Defeasible logic can resolve conflicting information by allowing the prioritization of rules through the superiority ($\succ$) relation. E.g. $r1 \succ r0$ means that rule $r1$ takes precedence over rule $r0$. This can be used to resolve conflicts between rules, or to determine the applicability of a rule in a given context.

Finally, DDL takes into account deontic properties such as Obligation (O), Permission (P) and Prohibition (F) and their relationships in SDL. For example, as to the attribute `ExclusiveSupplier`, the Prohibition against acting an exclusive supplier is equivalent to the Obligation not to act as an exclusive supplier.

A is concerned with the limiting the number or range of buyers and suppliers.

$$[\text{F}]\text{ExclusiveSupplier} \equiv [\text{O}]\neg\text{ExclusiveSupplier} \tag{6.15}$$

Thus, the rule that disallows exclusive suppliers (subject to the exceptions for natural monopolies) can expressed as:

Consider looking at the rest of Category A in the CIA guidelines.

$$\emptyset(\text{Empty Set}) \Rightarrow [\text{F}]\ \text{ExclusiveSupplier} \tag{6.16}$$

$$(\text{Sector\_AllowsNaturalMonopoly(Acme Inc.)}) \Rightarrow [\text{P}]\ \text{Exclusive Supplier} \tag{6.17}$$

6.1.5 Reified I/O Logic

Reified I/O logic is a formalism for representing norms in textual form. It combines Input/Output (I/O) Logic from Markinson and Van der Torre (2000) with a reification-based logic (Hobbs and Gordon 2017)[33].

I/O Logic

I/O logic is a logical formalism developed by Makinson and Van der Torre (2000). An Input/Output system is a set of if-then rules in the form of (a, b), called Input/Output pairs or Input/Output generators.[34] Where:

- a - Is the given input, or the Left Hand Side (LHS) of the pair;
- b - Is the returned output, or the Right Hand Side (RHS) of the pair

Both the LHS and RHS of a given Input/Output pair are whole formulae in some object logic with language L. Thus given a set A of formulae in an object logic, $C_n(A)$ is the set of all formulae derived from A through the inference rules of the logic ($C_n(A) = \{a \in L : A \vdash a\}$). In an input/output system: 1. We provide an input (if, LHS); 2. It goes through the axioms (or derivation rules) of the

[33] Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5689; Gordon et al has built an extensive index of predications (possible modifiers of subjects), encoding them based on the ISO/IEC 2407 the Common Logic Standard. Andrew S. Gordon and Jerry R. Hobbs. *A Formal Theory of Commonsense Psychology: How People Think People Think*. 1st ed. Cambridge University Press, Aug. 31, 2017. ISBN: 978-1-107-15100-0 978-1-316-58470-5. DOI: 10.1017/9781316584705. URL: https://www.cambridge.org/core/product/identifier/9781316584705/type/book (visited on 12/05/2024), pp. 54–55.

[34] Robaldo et al., "Formalizing GDPR Provisions in Reified I/O Logic", at 6-7.

system in order to return an output (then, RHS); 3. Finally, one can add axioms to the system in order to get different outputs to the system in order to get different outputs for different inputs (i.e. we are providing new operational meanings).[35]

Input propositions are not in general included among the outputs, and the operation is not in any way reversible.[36] I/O operations resemble traditional logical inference in that it derives conclusions (outputs) from premises (inputs) based on rules (the normative system). The normative system is not a collection of truths or propositions to be evaluated. Instead, it is a mechanism for transformation. It's a black box that takes in inputs(factual circumstances, context, conditions) and processes them to generate outputs (obligations, permissions, goals, desired states of affairs).

For this implementation, we adopt Robaldo's model of an I/O system that has two stages for constitutive and regulative norms:[37]The first set C is the set of constitutive norms. Every pair (x, y) is a standard first order implication $x \rightarrow y$; The inputs of C are the facts of the domain, and the output are the institutional facts. Based on Searle (1995),[38] C determines when something counts as something else in the domain. The output of C then becomes the input to two other sets of pairs, O and P where O is the set of obligations and P is the set of permissions.[39]

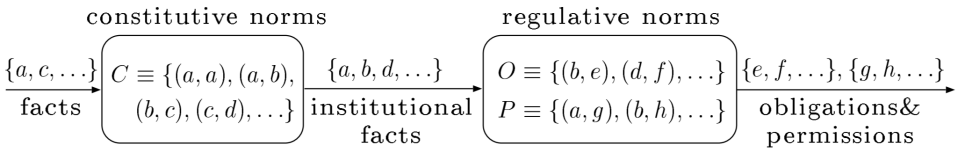
1. A pair $(x, y) \in O$ means "given x, y is obligatory";

2. A pair $(x, y) \in P$ means "given x, y is permitted".

Given different axioms on C, O, and P we can make a computation as to what is statements are obligatory, forbidden, what obligations have been fulfilled, violated, and what are still in force.

[35]Robaldo et al., "Formalizing GDPR Provisions in Reified I/O Logic", at 7.
[36]Ibid., at 6-7.
[37]Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5689.
[38]John R. Searle. *The Construction of Social Reality*. New York: The Free Press, 1995.
[39]Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5689.

Figure 6.2: 2-staged I/O logic system for constitutive and regulative norms(Robaldo, 2020)



While it enables us to model the basic contingent structure of norms, I/O logic is strictly propositional. That is, its basic components are whole propositions,whereas real world semantics contain "intra-sentence linguistic phenomena" - like named entities, anaphora, quantifiers".[40]

Reification

In encoding the components of a normative statement, the formalism relies on reification based on the system outlined by Robaldo.[41] Reification formalizes events and states to correspond to First Order Logic (FOL)terms (constants, variables, functional terms).[42] It goes beyond basic propositional form of FOL (which lacks expressivity),[43] and has the additional advantages avoiding the nesting of subformulae within complex operators (which allows us to represent complex linguistic phenomena with flat atomic FOL (reified) predicates in the ABox) while the definitions of these predicates are modeled as separate axioms in the TBox.[44] At the same time, it retains features from deontic logic and defeasible deontic logic, e.g., 1. deontic operators representing modalities that are involved - obligatory, permitted, prohibited; 2. non-monotonic operators for handling the role of

[40]Livio Robaldo and Xin Sun. "Reified Input/Output Logic: Combining Input/Output Logic and Reification to Represent Norms Coming from Existing Legislation". In: *Journal of Logic and Computation* 27.8 (Dec. 1, 2017), pp. 2471–2503. ISSN: 0955-792X, 1465-363X. DOI: 10.1093/logcom/exx009. URL: http://academic.oup.com/logcom/article/27/8/2471/3098296 (visited on 12/16/2024), at 2-3.

[41]Robaldo and Sun, "Reified Input/Output Logic"; Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML".

[42]Eventualities can be treated as if they were objects of human thoughts. As reified eventualities can act as parameters for predicates like *believe*,*think*, *want* Robaldo and Sun, "Reified Input/Output Logic", at 10-11.

[43]Livio Robaldo et al. "Compliance Checking on First-Order Knowledge with Conflicting and Compensatory Norms: A Comparison among Currently Available Technologies". In: *Artificial Intelligence and Law* 32.2 (June 2024), pp. 505–555. ISSN: 0924-8463, 1572-8382. DOI: 10.1007/s10506-023-09360-z. URL: https://link.springer.com/10.1007/s10506-023-09360-z (visited on 07/03/2025), at 506.

[44]Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5690.

defeasibility in normative reasoning.[45] In a strictly propositional scheme, we can only represent the premise (IF "Law allows single supplier") and the consequence (THEN "Rights are exclusive") of a normative statement as indivisible atomic components, and by default we cannot make further granular representations of their semantics without the involvement of complex nested structures. Reified I/O logic allows us to treat the predicates and logical connectives (and even operators) as the classes and properties of the ontology, so that they can have their own semantics.

Predicates and eventualities

Following Gordon and Hobbes,[46] verbs and properties are not represented as standard predicates $P(x, y)$. Instead, these are reified into terms denoting events or states. The FOL statement $gives(John, book, Mary)$ becomes a flat conjunction of atomic predicates over a reified variable e (the eventuality of giving):

$$(give' \ e \ John \ book \ Mary) \wedge (Rexist \ e)$$

Where:

- $give'$ - the primed predicate relating the event term e to its semantic roles.
- $Rexist$ - a meta-predicate asserting that event e really exists in the timeline or holds true.

States and events denoted by these terms are treated like things in the world. Thus, “eventuality" denotes both the reification of a state as well as that of an event or action.[47] In their system every FOL predication, e.g. $(dominant \quad \text{Meta})$, which asserts that Meta is dominant firm may be associated with another FOL predication, $(dominant' \quad e_d \quad \text{Meta})$ where e_d is a reified eventuality - i.e. the

[45] Robaldo et al., “Compliance Checking on First-Order Knowledge with Conflicting and Compensatory Norms”, at 506.

[46] Gordon and Hobbs, *A Formal Theory of Commonsense Psychology*.

[47] Robaldo and Sun, “Reified Input/Output Logic”, at 3.

term e_d is the **reification** of Meta's status as a dominant firm.[48]

Recursive Reification

From the reification of the state of being dominant, other predications may be applied to e_d, and then recursively reified to express more elaborate semantics. For example, the statement that Meta intends to be dominant can be encoded as:

$$(intend'\ e_i\ Meta\ e_d) \wedge (dominant'\ e_d\ Meta) \tag{6.18}$$

The fact of Meta's intention represented by e_i. This predication is applied to the previously defined e_d, the reified eventuality of being dominant. We can then instantiate the eventuality of Meta intending to obtain a dominant position through a special predication called $RexistAtTime$, providing for its real existence within a time context t:

$$(RexistAtTime\ e_i\ t) \tag{6.19}$$

This allows us to make a distinction between actually being dominant, and only intending to be so - since only e_i has a real existence for a given time. The final representation of the statement that Meta intends to be dominant, without making any inference about it being actually dominant, can be written as follows:

[48] Hobbs logical framework contemplate two sets of logical predicates working in parallel: primed and unprimed. Unprimed refers to the standard FOL predicates, e.g. $(give\ a\ b\ c)$ asserts that a a gives b to c; Primed represents reification of the corresponding unprimed relation, e.g. $(give'\ e\ a\ b\ c)$ says that e is a giving event by a of b to c. ibid., at 10.

$$(RexistAtTime\ e_i\ t) \wedge (intend'\ e_i\ Meta\ e_d) \wedge (dominant'\ e_d\ Meta) \tag{6.20}$$

The extent of the use of reification is such that even boolean operators can be represented as eventualities. For example, $(not' \quad e_1 \quad e_2)$ is used to assert that e_1 is the eventuality of e_2's non-existence.[49] The same applies to boolean operators for *and* as well as *or*.

These statements based on reified logic can then be used to model normative statements with the standard structure of I/O logic. In I/O logic, each normative statement is represented by input/ouput pairs (x, y) with x at the left hand side of the comma representing the precedent, and the right hand side y representing the consequent of the normative conditional.[50] The output is then considered as an input to further processing as part of any of sets C, O and P (constitutive rules, obligations, and permissions).[51]

Constitutive and Regulative Norms

In Reified I/O logic, legal reasoning is modeled through two sequential "black boxes" to distinguish between Constituent Norms (which define institutional concepts) and Regulative Norms (which prescribe behavior based on these concepts)

[49] *not'* is not a boolean operator, but is an FOL predicate that describe (through relations) 2 eventualities, one of which is the negation of the other. Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5689.

[50] Robaldo et al., "Formalizing GDPR Provisions in Reified I/O Logic", at 6.

[51] It is based on established distinctions between regulative norms and constitutive norms. (Searle, 1995). 1. Regulative norms - are obligations and prohibitions (the deontic statements) while 2. Constitutive norms - definitions of the concepts used in regulative norms Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5689; Searle, *The Construction of Social Reality*.

Constitutive Norms Citing Searle (1995)[52] Boella insists that encoding law requires not just regulative norms (obligations, prohibitions, and permissions), but also constitutive norms.[53] These define that something counts as something else for a given institution.

Constitutive norms have to be accounted for since often, regulative norms are contingent on pre-existing definitions. New situations will require reformulation of conditions. Instead of introducing new norms or changing existing ones, regulative norms could refer to abstract concepts only, factoring out specific cases. Constitutive norms can define more abstract, institutional classification, and changes to conditions of norms can be treated as changes to the more abstract classification.Thus, constitutive norms provide a system for specifying:1. How the regulative norms can be changed (through new norms and new institutional categories); as well as 2. Who gets to introduce valid, binding changes[54]

The substance of constitutive norms is based on the distinction between brute facts (e.g. this mountain or ocean or molecule exists, and these exist regardless of human opinion or agreement), and institutional facts (e.g. the state exists, money exists because humans agree that they exist). The classic example of constitutive rules are the rules of a game, such as chess. These rules don't just simply regulate an antecedent activity called playing chess. Instead, it creates the possibility of an activity called chess. For example the rule defining the bishop's movement of the bishop piece is not a constraint on some pre-existing way of moving bishops. The rule is what makes a particular piece of wood a bishop's move in the first place. Of course, constitutive rules not just about games. These rules can construct institutions: Marriage, money, property, government. A bill is just a piece of paper (brute fact) until a system of constitutive rules endows that piece of paper a special status (as "money"). This then creates new possibilities for action: buying, selling, investing.

[52] Searle, *The Construction of Social Reality*, According to Searle, constitutive norms do not merely regulate and antecedently existing activity (e.g. chess), but instead creates the possibility or defines the activity (e.g. the rules of chess). Other systems of constitutive rules include marriage, money, the acts and expectations of promising.

[53] Boella and van der Torre, "Regulative and Constitutive Norms in Normative Multiagent Systems", at 255.

[54] Ibid., at 255-256.

Constitutive rules bridge the ontological gap between brute facts and institutional facts. Through constitutive rules, we can impose status and functions upon objects and behavior. The transformation of brute facts into institutional facts through constitutive rules is made possible through the human capacity for language and collective agreement. Thus: While we can train a dog to follow regulative norms (if I whistle, come to me), but we cannot really teach it to vote in an election, or play chess as such (even though it may be capable of repeating the necessary actions mechanically). Because these activities are constituted by rules that require **symbolic representation**. Institutional reality (as created by the constitutive rules) is sustained by collective acceptance and agreement. Rules are in force because the relevant community recognizes and acts upon them[55]

Regulative Norms Regulative norms , on the other hand, regulate antecedently existing forms of behavior (e.g. rules of polite table behavior regulates eating, but eating exists independently of those rules).[56] Regulative norms govern the behavior of things that exist independently (or antecedently) of the norms themselves.These can be expressed as commands ("Do X") or as conditional imperatives ("If Y, Do X").

The oft-cited classic example of regulative norms from Searle are traffic rules, e.g., the drive on the right side of the road, or stop at a red light.The activity of driving existed (logically and practically) prior to the existence of traffic rules. The regulative norms of traffic rules merely imposed a standard on pre-existing behavior.[57]

6.1.6 SHACL as Legal Encoding Language

Selection Criteria for Encoding Language

Andersson asserts that most software tools (e.g. most modern, general purpose programming languages), while usable, may nevertheless be overscoped for implementing the requirements of

[55] Searle, *The Construction of Social Reality*.
[56] Boella and van der Torre, "Regulative and Constitutive Norms in Normative Multiagent Systems", at 256.
[57] Searle, *The Construction of Social Reality*.

a computational law system. It would be more efficient (cost-benefit wise) to develop and use domain-specific languages for computational law.[58] Such a domain-specific language for representing the law should have the following features:[59]

1. Can accommodate semantically-rich rulesets within a context of formal modeling of behavior.
2. Have grounding in logic, since we are concerned not just with representing knowledge, but performing (logic = programmable = computable) automated analysis on that knowledge.
3. Capability to represent sequences of actions, and behavioral constraints (such as law) upon those actions.

To this, Andersson provides additional requirements such as 1. **Independence**, which is met if the system does not require additional tools or languages in order to meet computational law functions; 2. **Correctness**, which is met if and only if its outputs from given inputs conform with the responses expected by legal professionals. Correctness need not mean "unambiguous" - Systems can either return correct but ambiguous results, or limit coverage of the programming language domain to those matters that are not ambiguous; 3. **Expressiveness** - refers to the "extent that we can translate ordinary legal documents into the language system without sacrificing correctness" (A system is unexpressive if ordinary legal writing cannot be correctly translated into it); 4. **Determinacy** - is met if the system or language always generates the same responses to the same inputs; 5. **Usability** - is met if legal professionals can represent legal texts and derived legal solutions without difficulty (or at least, no greater difficulty than expected in the practice of law)[60]

Several languages meet the above criteria and were considered for implementing the programmatic account of the normative encoding. Since Robaldo's work on Reified I/O Logic to represent the GDPR employed Legal Rule ML as its programmatic layer, the author originally considered using the same technology. The author also considered other representational frameworks, such the

[58] Andersson, "Computational Law: Law That Works Like Software".
[59] Genesereth and Love, "Computational Law", at 206.
[60] Andersson, "Computational Law: Law That Works Like Software", at 22-23.

Rule Interchange Format as well as Legal Knowledge Interchange Format. For the reasons to be elaborated below, the author has settled on the Shapes Constraint Language. Unlike the case for OWL, there is no publicly available reasoning engine engine for LegalRuleML. Although OWL and LegalRuleML can interoperate with each other, an OWL reasoner (which is based on Description Logic) cannot be used to make inferences on a LegalRuleML graph. SHACL is integrated into the RDF ecosystem - with several reasoners available through Protégé - the same environment used to design the ontology. This will make it easier to integrate the normative reasoning with the semantics - i.e. ensuring that the objects subject to normative constraints are those defined in the ontology, and that normative constraints take into account ontological definitions.SHACL also has the same RDF-based syntax as OWL to facilitate coding, and is expressive enough to model elements from Robaldo's version of Reified I/O logic - i.e. it can accommodate both deontic logic and defeasibility. SHACL strikes a balance in computational modelling of norms: 1. It has sufficient grounding in the legal domain, so that it can be understandable to the domain experts; 2. At the same time, it can be formalized in a way that would allow computation.[61]

Overview of SHACL

SHACL - Aimed at producing a language for defining structural constraints on RDF graphs. SHACL has two parts, both of which will be used for modelling the norms of the OECD Guidelines:[62]

1. SHACL Core - A core RDF vocabulary to define common shapes;

2. SHACL-SPARQL - An extension mechanism that integrates SPARQL.[63]

[61] Tara Athan et al. "OASIS LegalRuleML". in: *Proceedings of the Fourteenth International Conference on Artificial Intelligence and Law*. ICAIL '13: International Conference on Artificial Intelligence and Law. Rome Italy: ACM, June 10, 2013, pp. 3–12. ISBN: 978-1-4503-2080-1. DOI: 10.1145/2514601.2514603. URL: https://dl.acm.org/doi/10.1145/2514601.2514603 (visited on 12/16/2024), at 6.

[62] José Emilio Labra Gayo et al. *Validating RDF Data*. Synthesis Lectures on the Semantic Web: Theory and Technology #16. San Rafael, California: Morgan & Claypool Publishers, 2018. 1 p. ISBN: 978-1-68173-164-3 978-1-68173-165-0, at 119.

[63] SPARQL is a recursive algorithm which stands for SPARQL RDF Query Language, a query language used to retrieve data from any RDF-based data structures.

SHACL groups the information and constrains that apply to data nodes into some constructs called shapes. A **shape** is a collection of constraints to be applier to some node in an RDF graph and if it is closed every property attached to that node must be included in the shape.[64]

SHACL represents shapes as a conjunction of constraints that nodes must satisfy. A shapes graph contains shapes definitions that will be passed to the SHACL validation process. Shapes can be reused, by referencing them from other shapes graphs with the predicate `owl:imports`.[65] A SHACL processor checks each of the constrains and returns validation errors for every constraint that is not satisfied. If no error is reported, it is assumed that the RDF graph has been validated.[66] The SHACL processor expects two inputs: 1. A data graph that contains the RDF data to validate (in this case, an OWL representation of the law to be evaluated for competition impact); 2. A shapes graph that contains the shapes (in this case, SHACL code representing the norms of the OECD Guidelines).[67] When an RDF graph conforms to a shapes graph, SHACL processors return a validation report with no errors.[68] The validation report is defined as an RDF graph with the following structure: If the data graph conforms to the shapes graph, the report contains a `sh:conforms` declaration with the value `:true`. On the other hand, if the data graph does not conform to the shapes graph, the validation report will have a value `:false` for the property `sh:conforms` and a set of validation errors of type `sh:ValidationResult` linked by the property `sh:result`.[69]

How SHACL Validates Data

It is beyond the scope of this chapter to provide an explanation of the internals of SHACL and its validation process. To provide an overview of the process, however, we outline the broad steps below.

1. The SHACL engine takes in an RDF data graph, which can represent, in this case, the norm

[64] José Emilio Labra Gayo et al., *Validating RDF Data*, at 44-45.
[65] Ibid., at 125.
[66] Ibid., at 121.
[67] Ibid., at 122.
[68] Ibid., at 121.
[69] Ibid., at 126.

we wish to evaluate, and a SHACL shapes graph that can specify the constraints that the said norms should comply with (in this case A1 of the OECD Guidelines);

2. First, we specify in the SHACL shapes graph the subset of nodes in the data graph that are supposed to conform with particular shapes. Such focus nodes are defined in the shapes graph through the specification of target nodes. SHACL can specify targets by defining all instances of a particular RDF class, an explicity named RDF resource, or any node that acts as a subject or object for a specific predicate;

3. Once the engine pairs a target node with a shape, it compares this target against either node shapes (which validates attribute of the focus node itself), or property shapes (which validate the properties linked to the focus node);

4. Since we will need to evaluate individual attributes contained in target nodes, we will use property shapes, the SHACL engine will need to traverse a path from the focus node to the actual values that need to be tested. Once the specified value nodes are in focus (via the target node declarations), the engine traverses from the focus node to the actual values that need to be evaluated. This traversal is directed by the parameters in the ‘sh:path‘ directive. The resources it lands on at the end of this traversal are called value nodes;

5. The engine then checks whether the value nodes in the data graph complies with the constraint components specified in the shapes graph. SHACL has an extensive library of built-in constraint components, ranging from simple cardinality constraints to regular expressions of complex logical conditions. The engine evaluates every value in the captured target nodes. If a constraint is not complied with, a violation is triggered and is logged as an error (`sh:conforms false`) in a separate RDF validation report.

6.2 Formalized Representations for Section A1

In the subsequent section we demonstrate how the normative encoding applies to a specific norm in the OECD Guidelines (A1). To achieve this, we will proceed step-by-step view from its reified IO logic encoding, to its SHACL encoding.

A1 provides that laws which allow exclusive rights to suppliers ought to be flagged for further assessment. This can be further formalized as follows: If there is a law that grants a supplier exclusive rights to the provision of goods and services, then a policy maker should flag the law and subject it to further competition assessment. That is, for every provision of law p and every instance of time t, if at t, x grants exclusive rights, then a policy maker x must conduct further assessment, where e_1 is the grant of exclusive rights, and e_c. This contains a single factual condition (presence of exclusivity), leading to a single obligation (to conduct further competition assessment).

Based on the encoding outlined by Robaldo this translates to:

$$
\begin{aligned}
&\forall p\ \forall t\ (\\
&\exists e_1\ \exists s\ \exists g[(RexistAtTime\ e_g\ p\ s\ g) \wedge \\
&(provision\ p) \wedge (supplier\ s) \wedge (goodsOrServices\ g) \wedge \\
&(grantsExclusiveRights'\ e_g\ p\ s\ g)], \\
&\exists e_c\ \exists_a\ \exists x[(RexistAtTime\ e_c\ t) \wedge \\
&(policyMaker\ x) \wedge (assessment\ a) \wedge \\
&(conductAssessment'\ e_c\ x\ a\ p)] \\
&) \in O
\end{aligned}
\tag{6.21}
$$

As was the case before, standard relations are transformed into primed predicates that take the eventuality as their first argument. For example, the action of a provision p granting exclusive rights to a supplier s for goods g is reified into the eventuality e_g via the primed predicate so that this eventuality can be treated as a specific entity.

$$(grantsExclusiveRights' \ e_g \ p \ s \ g)$$

$RexistAtTime$ indicates the eventualities that are asserted to be actually occurring - the fact that exclusive rights are being granted to the supplier e_g (on the LHS), and the fact that the conduct of an assessment as the consequence (on the RHS). Note that standard unprimed predicates (e.g. $provision \ p$) are basic class restrictions and do not require to be instantiated by $RexistAtTime$. The special predicate $RexistAtTime$ is used to explicitly declare that the eventuality actually exists in the real world at time t.

In order to keep individual formulas readable, as well as help with legibility and maintainability of the knowledgebase, Reified I/O Logic enables the distinct encoding of constitutive rules and regulative rules.

Separating Constitutive and Regulative Norms As discussed above, constitutive rules define the creation of institutional facts. Such rules do not regulate existing behavior but rather create new abstract legal categories, establishing "x counts as y in context c". In Reified I/O Logic constitutive norms act as the first "black box" assigned to set C, that takes brute facts of the world as input and produces "institutional facts" as output.[70]

Regulative rules, on the other hand, dictate what behavior is required, prohibited, or allowed. In Reified I/O Logic, these form the secon "black box", assigned to the set of obligations O or P for permissions, taking the institutional facts generated by the constitutive rules as input and producing normative directives as outputs.

[70] Robaldo and Sun, "Reified Input/Output Logic".

Application of the distinction to A1: A1 evaluates whether a legal provision "grants exclusive goals or services". According to the OECD Guideline, if this is true, then the provision "limits the number or range of suppliers", and consequently, a competition assessment should be conducted.

First, we define "what counts as what". The brute facts of granting exclusive rights to a supplier logically triggers the institutional fact that the provision limits the number or range of suppliers.

$$\begin{aligned} &\forall p\ \forall t(\exists e_g\ \exists s\ \exists g[(RexistAtTime\ e_g\ t) \wedge \\ &(provision\ p) \wedge (supplier\ s) \wedge (goodsOrServices\ g) \wedge \\ &(grantsExclusiveRights'\ e_g\ p\ s\ g)], \\ &\exists e_A[(RexistAtTime\ e_A\ t) \wedge (limitsSuppliers'\ e_A\ p)]) \in C \end{aligned} \tag{6.22}$$

The rule dictates that if the brute fact e_g exists in input (left-hand side), the system must output the institutional fact e_A (the provision p limiting supplier) on the right-hand side.

Next, the institutional fact generated in Step 1 is fed into the second deductive black box that evaluates what is obligatory. The OECD Guidelines prescribes an assessment when a provision limits suppliers, triggering the latter as an obligation.

Finally, the rule is assigned to the set C, designating it as a constitutive definition within the TBox of the ontology.

The output e_A from the constitutive rule becomes the factual input (left hand side) of the prescriptive rule. If e_A really exists, the rule outputs a new eventuality e_c which represents the policymaker x conducting a competition assessment on provision p.

By assigning this rule to the set O (the ABox), the logic designates the output e_c as an obligation (e.g. if the right hand side is not met, then the prescriptive rule representing A1 is considered violated)

$$\begin{aligned}
&\forall p\ \forall x \forall t(\exists e_A[(RexistAtTime\ e_A\ t) \wedge \\
&(policyMaker\ x) \wedge (provision\ p) \wedge \\
&(limitsSuppliers'\ e_A\ p)], \\
&\exists e_c\ \exists a[(RexistAtTime\ e_c\ t) \wedge \\
&(assessment\ a) \wedge \\
&(conductAssessment'\ e_c\ x\ a\ p)]) \in O
\end{aligned} \tag{6.23}$$

Quantifies manage variables across the I/O structure. Universal quantifiers ($\forall$) prepare variables that occur. in both the LHS input and the RHS output. In this case:

- $\forall_x$ - For every law x (Or $\forall p$, for every policymaker);
- $\forall_t$ - For every instance of time$t$.

Quantifiers Universal quantifiers, which expresses that the property or relations subsequently hold for any specified member - in this case laws and instances of time - within the domain.

- $\exists e_g$ - There exists the eventuality of being granted an exclusive right
- $\exists e_e$ - There exists the eventuality of being an exclusive right
- $\exists e_r$ - There exists the eventuality of being a right
- $\exists e_f$ - There exists the eventuality of being flagged

These are existential quantifiers (prefixed with $\exists$) which is used to express that there is at least one value for which the given predicates hold true. These preface variables that only appear on one side of the the I/O structure (e.g. suppliers, goods)

NOTE - This ha to be updated based on the latest formula encoding

Deontic Value as Obligation $\in O$ is required at the end of the formulation because the rule maintains that an action (conducting assessment) is obligatory when the input facts are met, both the LHS and RHS are assigned to the set of obligatory norms, denoted thtoughn $\in O$.

This allows the general obligation to trigger by default without requiring an exhaustive inquiry on the absence of every possible exception (See [[Robaldo2020Formalizing]])

$$
\begin{aligned}
&\forall p\ \forall t(\\
&\exists e_g\ \exists s\ \exists g[(RexistAtTime\ e_g\ t) \wedge \\
&(goodsOrServices\ g) \wedge \\
&(grantsExclusiveRights'\ e_g\ p\ s\ g) \wedge \\
&naf(exceptionNaturalMonopoly\ e_g)], \\
&\exists e_c\ \exists a\ \exists s)[(RexistAtTime\ e_c\ t) \wedge \\
&(policyMaker\ x) \wedge (assessment\ a) \wedge \\
&(conductAssessment'\ e_c\ x\ a\ p)] \\
&) \in O
\end{aligned}
\tag{6.24}
$$

Next, we define the specific condition under which the exception is triggered. If the supplier s involved in the exclusive rights eventuality e_g is a natural monopoly, the exception becomes true. This is encoded as a constitutive rule, as it defines "what counts as what":

$$
\begin{aligned}
&\forall e_g \forall t(\\
&[(RexistAtTime\ e_g\ t) \wedge \\
&(provision\ p) \wedge (supplier\ s) \wedge \\
&(goodsOrServices\ g) \wedge \\
&(grantsExclusiveRights'\ e_g\ p\ s\ g) \wedge (naturalMonopoly\ s)], \\
&(exceptionNaturalMonopoly\ e_g) \\
&) \in C
\end{aligned}
\tag{6.25}
$$

In the default scenario: If a legal provision grants exclusive rights to a standard (i.e. not a natural monopoly) supplier, there is no evidence that the supplier is a natural monopoly. Therefore, $exceptionNaturalMonopoly$ cannot be derived as true. Because it is unknowm, $naf(exceptionNaturalMonopoly\ e_g)$ evaluate to true. All conditions on the LHS are met, and the system successfully outputs the obligation to conduct an assessment.

If, on the other hand, the system is given facts proving the suppler is a natural monopoly, the constitutive rule above is triggered and establishes that $exceptionNaturalMonopoly$ is true. Consequently, $naf(exceptionNaturalMonopoly\ e_g)$ in Step 1 evaluates to false - this blocks the LHS of the rule, meaning the obligation to conduct a competition assessment is never generated.

6.3 Automated analysis and evaluation

Move to chapter 8

The canonical approach requires evaluation of the relevant laws for features that match a predetermined list of factors (usually based on the economics literature). It relies on both a reading of the text, and the lawyer's training on how the text is most likely interpreted and enforced. What

usually happens, based on the recommendations of these guides, is an appeal to the lawyer's intuition as to the intent and consequences of the legal text. Some of these guides suggest, to balance out the inherent subjectivities in that determination: Consulting other stakeholders (regulators and industry stakeholders). While this cross analysis might go a long way towards making the conclusions less stilted, there is still no proof of work that can be shared and independently studied, changed, and evaluated. We should be able to rely on a transparent chain of reasoning proceeding from plausible assumptions into consistent propositions, that can be shared, analyzed, built on top of each other.

Once we have the rules encoded, the goal is to perform automated evaluations. We can look for internal inconsistencies, or gaps in the coverage of industry entities and transactions. Then we can compare one set of rules - such as the legislation under competition impact assessment, with the standards set by the economic literature, or the competition authority, or international organizations. Once law is reduced to a formalized structure, then it becomes amenable to direct comparison - for finding difference and inconsistency. Unlike intuitive assessments, though, the reasoning process is exposed from the start - the assumptions are provided (or at least very easy to look up), and each step towards the conclusion is available for proof.

Ontologies and inference rules can be combined into network structures, and it is possible to compare network structures - i.e. to what extent these structures are similar or different. But beyond some of the more obvious methods, this work will explore two pathways that will enable computers to compare and evaluate the encoded rules: 1. Argumentation frameworks and 2. Propositional networks.

The first takes into account the dialectic nature of arriving at a legal determination. Conclusions about law are often only arrived at after an argument - one side presents a plausible reading of the law, another counters with a supposedly better reading of the law, or evidence of factual circumstances that would make the law inapplicable, or a higher law.[71] The initial proponent could counter, and

[71] See generally Frans H. Van Eemeren et al. *Handbook of Argumentation Theory*. Dordrecht: Springer Netherlands, 2014. ISBN: 978-90-481-9472-8 978-90-481-9473-5. DOI: 10.1007/978-90-481-9473-5. URL: https://link.springer.

on and on until the arguments are exhausted and a decision has to be made by some process and standard. In the computational law field, there are so called argumentation frameworks. These are tools for modeling both rules and facts into arguments. Normative claims can be encoded just like rules, while the facts embodied in knowledge graphs can serve as evidence, or a warrant that either supports or undercuts a claims. In order to be processed an argumentation framework, we need to add information as to how all the claims and warrants relate to each other - either supporting or attacking. A reviewer can set the burden of proof, the weight of different kinds of evidence, and the standard required for an argument to prevail over the other.

Another method to be explored is through propositional networks. Propositional networks are an extension of game theory.[72] It is used in artificial intelligence, used for playing games and programming logic. Under this approach, entities and transactions can be modeled as they are in a knowledge graph - related to each other through states, attributes, and transactions. Unlike the static representation of knowledge graphs, however, propositional nets allow us to model transitions in both entities and relationships that can be caused either by constraints or actions - which can be provided by law. Propositional networks can be used to model the behavior of entities and transactions over time, and how they interact with each other.

The approach should combine the norms in our deontic propositions with the structured information in a knowledge graph, such that the norms can interact with the semantic information. Because the law can assume that the [Driver] is an adult and is licensed, and if neither of those are true, then a different set of norms apply. At the same time, a state of [Violation] would mean that the status of [Driver] could be modified i.e., suspended or annulled.

com/10.1007/978-90-481-9473-5 (visited on 06/20/2023).

[72] See Michael Genesereth and Michael Thielscher. *General Game Playing*. Red. by Ronald J. Brachman, William W. Cohen, and Peter Stone. Synthesis Lectures on Artificial Intelligence and Machine Learning 24. Morgan & Claypool Publishers, 2014.

Chapter 7

Description of the Encoded Knowledge Base

New Chapter 7 - Mainly concerned with combined ontological and normative encoding of A1

7.1 Description of the Encoded Knowledge Base

Consider discussing - Is the CIA process really computable? What are the limits?

This chapter covers a description of a knowledgebase,[1] derived from a computational encoding of items A1-5 of the OECD Competition Impact Analysis Guidelines (“OECD Guidelines"). Focusing on A1 as an example, we will discuss the ontological account of the encoding, followed by a normative account.

7.2 Logical Account of the Encoding

Combining these insights, we can now model a hypothetical rule which states that: "Firms that are dominant and abuse their dominant position must be fined". In normative conditional form:

> IF a firm x is dominant and is abusing its dominant position
> THEN x paying a fine is OBLIGATORY. (7.1)

[1] OWL source code of the knowledge base is accessible through https://github.com/emersonbanez/dissertation

The corresponding Reified I/O formula, with generalizations based on First Order Logic, is as follows:

$$\begin{aligned}
&\forall_x \forall_t (\\
&\exists_{e_a} \exists_{e_{ab}} \exists_{e_d} [(RexistAtTime\ e_a\ t) \wedge \\
&(and'\ e_a\ e_{ab}\ e_d) \wedge (abuse'\ e_{ab}\ e_d) \wedge \\
&(dominant'\ e_d\ x)], \\
&\exists_{e_p} [(RexistAtTime\ e_p\ t) \wedge \\
&(pay'\ e_p\ x\ Fine)]) \in O
\end{aligned} \tag{7.2}$$

7.3 Design Overview

7.4 Encoding of CIA Guidelines - Checklist A

The rules in Checklist A are concerned with legal constraints that limit the number and range of suppliers, which creates the risk that market power will be created and reduce competitive rivalry.

7.4.1 A1 - Grant of exclusive rights

The text of A1 of the OECD guidelines states that a law should be flagged if it "Grants exclusive rights for a supplier to provide goods or services". Exclusive rights are sub-optimal from a competition perspective because they are the "ultimate barrier to entry" and can lead to monopoly pricing. The exclusive supplier, once entrenched, may also be difficult to regulate for the purpose (in terms of limiting market power and protecting public welfare).[2] The rule embedded in this text can be expressed as a normative conditional statement:

[2] OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, at 10.

Nouns	Adjective	Verb	Object
Law		grant - grants, is_granted_by	Rights
Rights	exclusive		
Seller (Supplier)		sell (provide) - sells, is_sold_by participates_in	Goods OR Service Market
Goods			
Services			
Market			
Buyer		participates_in	Market

Table 7.1: Classes and properties in Rule A1

IF a law grants exclusive rights for the provision of goods or services to a supplier, THEN the law **must** be flagged.

The statement yields several nouns that can be treated as classes, as well as verbs and adjectives that can be mapped into properties in the DL/OWL ontology, as laid out in Table 1.

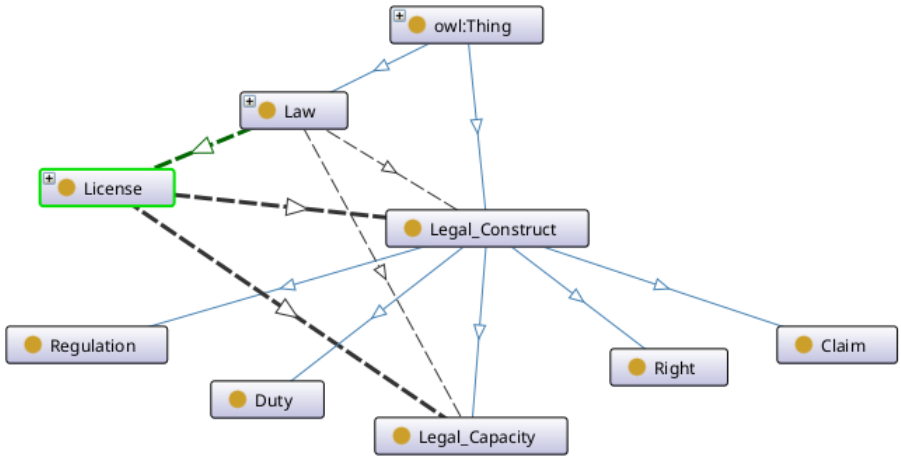
Model of Law and Rights The ontology adopts the LKIF distinction between: 1. The law, as embodied as an artifact, usually a `Legal_Document` (that is a subclass of `Document`, or $Legal\_Document \sqsubseteq Document$). It serves as the medium through which a `Legal_Expression` is made known.[3] Subclasses of this concept are organized based on the authority holding the norm, e.g. a `Legal_Document` can be `Statute`, `Treaty`, or `Contract` 2. The `Legal_Expression` itself, an abstract object which can contain propositions that define, or make deontic qualifications concerning a situation.[4] A law, as `Legal_Document` can contain one or several `Legal_Expressions`.

[3] "A legal document is a document bearing norms or normative statements. By virtue of this definition the norm-as-propositional-attitude is reified as norm-as proposition. In other words, the norm being expressed through the legal source is an expression of the propositional attitude." ESTRELLA Consortium. *Legal Knowledge Interchange Format (LKIF) - Core Ontology*. Version 1.1. 2008. URL: http://www.estrellaproject.org/lkif-core, at #Legal_Document; This corresponds with the concept of "Legal Authority" in FOLIO, defined as "Documents or publications that guide legal rights and obligations (e.g., case law, statutes, regulations, rules) or that can be cited as providing guidance on the law (e.g., secondary legal authorities)." The Institute for the Advancement of Legal and Ethical AI. *Federated Online Legal Information Ontology (FOLIO)*. FOLIO - Federated Online Legal Information Ontology (FOLIO). 2024. URL: https://folio.openlegalstandard.org/taxonomy/browse (visited on 04/28/2025), See "Legal Authority".

[4] "A norm is a kind of Qualification. A qualification which normatively qualifies some thing (i.e. some normatively qualified): i.e. a qualification which allows or disallows some thing." ESTRELLA Consortium, *Legal Knowledge Interchange Format (LKIF) - Core Ontology*, at #Norm; This corresponds with the FIBO concept of "law", defined as a

Each legal expression is the basis for each encoded formula in the knowledge base. A form of a norm is a `Right`, which qualifies a situation as something that a rights holder (usually some `Agent`) is entitled to.[5]

Figure 7.1: Class Hierarchy of Legal Concepts in A1



Persons Subject to A1 The rule contemplates several `Persons` who are the subjects of the rule - such as the `Seller`, `Buyer`, These participants are based on the LKIF `Agent` class, which is designed to be the basis of any actor with regard to an action. Based on the FOLIO ontology, a `Seller` is the person or organization that can, through a norm expressed in the law, have the right to supply the goods or services.[6] On the other side of that transaction is the `Buyer`[7]. Both `Seller` and `Buyer` are $\sqsubseteq$ *Persons* $\sqsubseteq$ *Agent*, with sellers also included in $\sqsubseteq$ *Organization* $\sqsubseteq$ *Persons*. Although sellers and buyers share the same ancestors (they are both persons and agents), the domain does not

"rule recognized by some community as regulating the behavior of its members and that it may enforce through the imposition of penalties" EDM Council. *The Financial Industry Business Ontology*. FIBO. Jan. 2020. URL: https://spec.edmcouncil.org/fibo/ (visited on 01/18/2024), See https://spec.edmcouncil.org/fibo/ontology/FND/Law/LegalCore/Law.

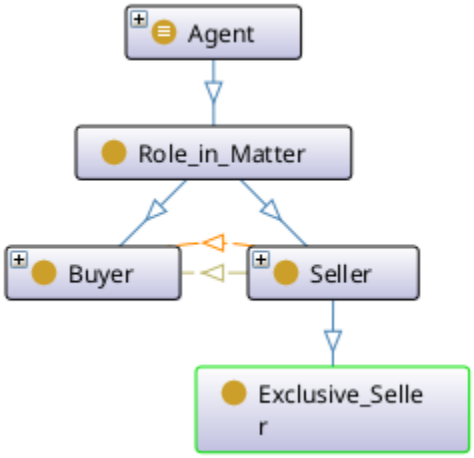
[5] "A right is the legal or moral entitlement to do or refrain from doing something or to obtain or refrain from obtaining an action, thing or recognition in civil society." ESTRELLA Consortium, *Legal Knowledge Interchange Format (LKIF) - Core Ontology*, See #Right; "entitlement (not) to perform certain actions, or (not) to be in certain states; or entitlement that others (not) perform certain actions or (not) be in certain states" EDM Council, *The Financial Industry Business Ontology*, See also.

[6] "A seller is a person, company or entity who sells a thing or property in exchange for other property (often money). One who disposes of a thing in consideration of money." The Institute for the Advancement of Legal and Ethical AI, *Federated Online Legal Information Ontology (FOLIO)*, See "Seller".

[7] "A buyer or purchaser generally means a person, company or entity contracting with a seller for services or merchandise to be provided or delivered for a named individual." ibid., See "Buyer".

contemplate that individuals can both be buyers and sellers, hence these two subclasses are **disjoint** ($Buyer \sqcap Seller \sqsubseteq \bot$) - that is, the intersection of these two classes will always be empty.[8]

Figure 7.2: Class Hierarchy of Participants in A1



Interaction of Concepts As discussed above, DL semantics can be elaborated not just through IS-A relationships between classes and subclasses, but through roles and their restrictions. Rule A1 contemplates that the sellers are in a position to sell `Goods` or `Services` to buyers. To represent these relationships, we can model the action of selling through a class called `Sale`. For every individual sales transaction, there is a seller, a good or service sold, and a buyer. The sale of goods can thus be represented through binary relationships with these other concepts:

$$\begin{aligned} Sale &\sqsubseteq \exists hasSeller.Seller \\ Sale &\sqsubseteq \exists hasGoodSold.Good \\ Sale &\sqsubseteq \exists hasBuyer.Buyer \end{aligned} \tag{7.3}$$

To further clarify the semantics of these relationships, we can also define the inverses of these roles, as reckoned from the objects of these transactions. Thus:

[8] $\bot$ is a special concept in DL, representing the class with no individuals as members

$$makesSale \equiv hasSeller^{-}$$
$$isGoodSoldIn \equiv hasGoodSold^{-} \tag{7.4}$$
$$participateInSale \equiv hasBuyer^{-}$$

Provide counterpart relationships linking each seller, good, and buyer, to a specific sales transaction.

Representing Exclusivity Each `Seller` participates in a `Market` where such seller can offer goods and services to multiple buyers. The rule in A1 prohibits (or at least negatively evaluates) a legal environment that allows any given seller to be the $Exclusive\_Seller$. We define an exclusive seller in terms of its participation in a $Monopoly\_Market$. That is, a market that only has one seller:

It's the law that creates the monopoly market. Why is there a need to examine the individual at this point?

$$Exclusive\_Seller \equiv Seller \sqcap (\exists participates\_in.Monopoly\_Market)$$
$$Monopoly\_Market \equiv Market \sqcap (= 1 has\_participant.Seller) \tag{7.5}$$

This is a very coarse, if not naive view of what it means to be an exclusive supplier for the purpose of competition impact analysis. There may be few purely monopolistic markets, and the OECD Guidlines are actually more concerned with manifestations of **market power**. Reckoned from the supplier side, this could mean the "the ability to profitably increase price, decrease quality, or decrease innovation relative to the levels that would prevail in a competitive market".[9] Although market power may elude precise legal definitions, there are assertions in the economic literature that can serve as the basis of an initial ontological encoding. There is the traditional line of thinking that evaluates market power bases on the extent of horizontal competition. That is, "competition from

How is this relevant? OECD CIA guidelines not really looking at market power (it is looking at the law, not the participants)

[9] OECD, *Competition Assessment Toolkit - Volume 1 (Principles)*, at 10, footnote 2.

established or potential rivals within a relevant market".[10] This is often focused whether or not any particular firm has the ability to set prices.This emphasis on the price setting power is incomplete, however and fails to give a satisfactory account of business models in a digital economy - not when the price is zero for the final consumer is zero for the final consumer, which in effect is subsidized by attention and data markets.[11] Lianos(2021), outlines possible alternative approaches based on either:1. The potential for coercion between market participants[12] and 2. A process-based definition, based on identifying specific situations of assymetry.[13]

Normative Encoding of Rule A1 The classes and properties in the ontology can then be used as the variables and predications in the norm, expressed as Reified I/O Logic formulae. In the case of Rule A1:

$$\begin{aligned}
&\forall_x \forall_t (\\
&\exists_{e_a} \exists_{e_{xs}} \exists_{s_1} [(RexistAtTime\ e_a\ t) \wedge \\
&(allows'\ e_a\ e_{xs}\ x) \wedge (exsupplier'\ e_{xs}\ s1) \wedge \\
&(supplier\ s_1)], \\
&\exists_{e_f} [(RexistAtTime\ e_f\ t) \wedge \\
&(flagged'\ e_f\ x)]) \in O
\end{aligned} \tag{7.6}$$

This translates to an obligation to flag a law if such a law allows any supplier to become an exclusive supplier (however it may be defined in the ontology).

This and other formulae needs to be elaborated and explained

[10] Ioannis Lianos and Bruno Carballa. "Economic Power and New Business Models in Competition Law and Economics: Ontology and New Metrics". In: *Center for Law Economics & Society Research Paper Series* (Mar. 2021). ISSN: 978-1-910801-37-6. URL: https://www.ucl.ac.uk/cles/research-papers (visited on 05/08/2025), at 6.

[11] Ibid., at 8.

[12] Coercion can still be vaguely defined, but Lianos proposes some possible operational bases: I.e., There is coercion when the choice forced upon the party coerced is such that she has no reasonable choice but to accept it. The absence of choice will require consideration of: 1. The relative bargaining position of the parties. 2. The history of their interactions. ibid., at 10.

[13] Ibid., at 11.

Modelling Exceptions The OECD guideline provide an exception to the negative evaluation of exclusive suppliers. That is, of the exclusive supplier is a natural monopoly. Assuming the concept can be encoded into the ontology, we can then model it as an exception to the normative encoding:

1. A law that allows a supplier to become an exclusive supplier ought to be flagged
2. BUT: If the exclusive supplier is a natural monopoly, then the law should not be flagged

In Reified I/O logic, exceptions are represented through Hobbs and Gordon's constructs for representing defeasibility: There are general predicates, and special "blocking" predicates that apply in the event of the exception. That is, the general inferences apply if more specific exceptions do not occur. The system for exceptions employ negation-as-failure - $naf(X)$, supported in LegalRuleML. That is, the predication $naf(X)$ is true when it cannot be derived that X is true (it is either false or unknown).[14] The exception, as the more specialized situation, is presumed to operate by default. Only when the exception does not occur will the general rule become applicable. For Rule A1, we can represent the exception as $naf(exceptionNaturalMonopoly\ s_1)$, which is true if the supplier s_1 is not a natural monopoly. Accounting for the exception, the encoding for A1 will now be:

$$\begin{aligned}
&\forall_x \forall_t(\\
&\exists_{e_a} \exists_{e_{xs}} \exists_{s_1} [(RexistAtTime\ e_a\ t) \wedge \\
&(allows'\ e_a\ e_{xs}\ x) \wedge (exsupplier'\ e_{xs}\ s1) \wedge \\
&naf(exceptionNaturalMonopoly\ s_1) \wedge (supplier\ s_1)], \\
&\exists_{e_f} [(RexistAtTime\ e_f\ t) \wedge \\
&(flagged'\ e_f\ x)]) \in O
\end{aligned} \tag{7.7}$$

[14] Negation as failure - A rule of inference where if a starting proposition P cannot be proven to be true based on the available knowledge and after an exhaustive search, then its negation (not true) is considered to be true. Contrast this with strong negation (or classical negation), where $notP$ can only be concluded if there is explicit evidence or a direct derivation proving that P is false. Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5693.

Modelling Interpretations Legal texts may contain ambiguous terms that can be subject to multiple, often conflicting interpretations. For example, we have previously discussed that in the context of competition law, "exclusivity" can be correlated with market power, which in turn can be viewed from multiple aspects:

- **Pricing Power View** - A firm has market power if it can set prices;
- **Coercion View** - A firm has market power if it can deny competitors reasonable choices;
- **Process View** - A firm has market power if it has bargaining power, especially related to the allocation of surplus.

Justify how this ir relevant to impact assessments

We can capture the proposition that exclusivity and pricing power are correlated through a constitutive rule:

$$\begin{aligned} &\forall_{e_{xs}} \forall_t (\\ &\exists_f [(RexistAtTime\ e_{xs}\ t) \wedge \\ &(exsupplier'\ e_{xs}\ f)], \\ &(PricingPower\ e_{xs})) \in C \end{aligned} \tag{7.8}$$

As in the case of exceptions, the representation of proposed interpretations is done through a special predicate $(assumption\ e_{xs})$, which is true if it can be assumed that the previous formula is valid:

$$\begin{aligned} &\forall_{e_{xs}} \forall_t (\\ &\exists_f [(RexistAtTime\ e_{xs}\ t) \wedge \\ &(exsupplier'\ e_{xs}\ f) \wedge (assumption\ e_{xs})], \\ &(PricingPower\ e_{xs})) \in C \end{aligned} \tag{7.9}$$

Differing interpretations as to the meaning of exclusivity can then be represented based on whether or not $(assumption\ e_{xs})$ holds or is negated. Thus, an interpretation that exclusivity should not be understood in terms of pricing power can simply negate the assumption:

$$\begin{aligned} &\forall_{e_{xs}} \forall_t (\\ &\exists_f [(RexistAtTime\ e_{xs}\ t) \wedge \\ &(exsupplier'\ e_{xs}\ f)], \\ &\neg(assumption\ e_{xs})) \in C \end{aligned} \tag{7.10}$$

Combining the mechanism for exceptions, we can model another possible interpretation where:

1. Exclusivity means pricing power in the general case;
2. Firms in the digital sector are an exception, where exclusivity means the ability to coerce possible competitors.

The general case is encoded as a constitutive rule where $(assumption\ e_{xs})$ and $naf\,(exceptionDigitalSector\ e_{xs}$ both hold true:

$$\begin{aligned}
&\forall_{e_{xs}} \forall_t (\\
&\exists_f [(RexistAtTime\ e_{xs}\ t) \wedge \\
&(exsupplier'\ e_{xs}\ f) \wedge naf(exceptionDigitalSector\ e_{xs})], \\
&(assumption\ e_{xs})) \in C
\end{aligned} \tag{7.11}$$

On the other hand, the exceptional case has additional predicates to reflect the situation where a firm is part of the digital sector:

$$\begin{aligned}
&\forall_{e_{xs}} (\\
&\exists_f \exists_t [(RexistAtTime\ e_{xs}\ t) \wedge \\
&(exsupplier'\ e_{xs}\ f) \wedge (partOf\ e_{xs}\ e_d) \wedge (digitalSector\ e_d)], \\
&(exceptionDigitalSector\ e_{xs})) \in C
\end{aligned} \tag{7.12}$$

Conclusion This chapter provided details on the formalization of a specific rule in the OECD Guidelines, using the methodologies outlined om the previous two chapters. Although both the ontological and normative layers of the encoding are implemented in software, a logical account was necessary to describe the structure and content of the encoding in a human-readable fashion, as well as demonstrate the feasibility of using formalizations for the domain of competition law.

The knowledge base, as of time of writing, encompasses both the ontological and normative encoding of rules A1 to A5 of the OECD Guidelines. From this point, further research can be performed to expand the coverage to the rest of the guidelines. Alternatively, future work can focus on enhancing the granularity and resolution of the encoding, through the definition of additional concepts, exceptions and interpretations.

Can be about:1. Limiting the license - size, capitalization requirements;2. A general approach may be too cumbersome. We can limit the analysis to a specific set of circumstances;3. See Prof. Soh's work on divorce rules;3. Some relevant variables: a.

Chapter 8

Analysis of the Encoded Structures

8.1 Legal Reasoning in General

In previous chapters, we have laid out some preparatory work for the encoding of the law: Terminological information, which can be read into a logical account (through Description Logic) and computationally encoded into an OWL ontology. The normative content of the law, on the other hand, has a logical account via Reified I/O Logic. This section examines the computational encoding of the norms, through the Shapes Constraint Language (SHACL).

The threshold test of competition impact assessment can be stated more explicitly as follows: Given a set of rules (i.e., the rules that cover an industry) - does it comply with or diverge from the idealized norm of the threshold test? Operationalizing the threshold testing problem into a computational concern:

1. Given a particular baseline norm, defined as a set of data constraints (or "shape") in SHACL; and

2. Given a particular rule being evaluated (encoded as a factual instance that assuming that the law was operational)

3. Is the rule compliant with the SHACL data constraints (i.e. does it have the same shape)?

8.1.1 Elaborating the Encoding of the Competition Norm

The logical encoding of A1 in Reified IO logic in (31) and (32) captures the main feature of the rule, but in its current state is not administrable and barely computational - we can either rely on user interaction for input as to exclusivity (which brings back discretion and frustrates the point) or just relying on defining an arbitrary cardinality threshold (which may be naive and unusable in actual competition law enforcement settings).

Robaldo (2020) suggests that Reified IO logic encodings can be made more specific by correlating them with court interpretations. As an example, he provides a possible correlation between Art. 33(2) of the GDPR and Section A 9.1 of the ISO/IEC 27018:2014 security standard.[1]

[1] Robaldo, Bartolini, and Lenzini, "The DAPRECO Knowledge Base: Representing the GDPR in LegalRuleML", at 5693.

Chapter 9

Normative Considerations

9.1 Normative Considerations for Adoption

9.1.1 The Landscape - Computational Law and Competition Enforcement

The previous sections outlined a workflow for the encoding of competition norms into a computational form, providing a test case from the OECD Competition Impact Assessment Guidelines. Specifically, Section A1, with correlations from Art. 102 of the TFEU was represented in the Shapes Constraint Language(SHACL). Then, the SHACL encoding was used to determine the compliance of norms encoded as RDF data.

The work is hoped to serve as proof-of-concept for computationally-assisted impact assessment of other laws, although it can also be used for compliance-checking of transactions. However, this should be considered within a background of existing technology. The market has seen a proliferation of specialized Legal AI tools - usually general purpose LLMs modified to make them more suitable for legal workflows.[1]

[1] Bob Ambrogi. *Legal AI Tools Show Promise in First-of-its-Kind Benchmark Study, with Harvey and CoCounsel Leading the Pack*. LawSites. Feb. 28, 2025. URL: https://www.lawnext.com/2025/02/legal-ai-tools-show-promise-in-first-of-its-kind-benchmark-study-with-harvey-and-cocounsel-leading-the-pack.html (visited on 12/11/2025), Some leading examples of such tools include:1. Harvey - An early leader in the field (funded by the Open AI Fund) and deployed in major firms like A & O Shearman. It can respond to complex natural language queries, 2. Westlaw's Co Counsel - is built on top of GPT and is capable of document review and deposition preparation (it is integrated into the

Many competition authorities worldwide have deployed (or are currently building) computational systems to assist in investigation and enforcement actions. Most of these systems adopt the connectionist appproach. One common application for these systems is the analysis of voluminous datasets in order to look for patterns that suggest anti-competitive behavior (e.g. bid rigging, collusion). Below is a partial table of some notable systems:[2]

Table 9.1: Computational Systems Used by Enforcement Authorities

Jurisdiction/Agency	**System Name**	**Description**
Brazil/CADE (Administrative Council for Economic Defense)	Project Cerebro	Data mines public procurement documents for signs of bid rigging Based on unsupervised machine learning, analyzing 40 separate databases (such as procurement data, tax records, corporate registries)
Spain/CNMC (National Markets and Competition Commission)	BRAVA (Bid Rigging Algorithms for Vigilance in Antitrust)	Used for the analysis of public tenders for signs of collusion. Based on supervised machine learning (trained on data sets where collusion was historically proven). Has provisions for explainability.

Westlaw ecosystem of tools), 3. Lexis+ - Westlaw's biggest competitor (in the legal research duopoly) also has a similar AI system, which promises reduced hallucinations by employing Retrieval-Augmented Generation to ensure that every text generated is tethered to documents from verified authors.

[2]Schrepel and Groza, *Computational Antitrust Worldwide: Fourth Cross-Agency Report*; See also OECD. *Data Screening Tools for Competition Investigations*. OECD Competition Law and Policy Working Papers 284. Oct. 18, 2022. DOI: 10.1787/4c5bbb9d-en. URL: https://www.oecd.org/en/publications/data-screening-tools-for-competition-investigations_4c5bbb9d-en.html (visited on 12/14/2025); OECD. *Detecting Cartels for Ex Officio Investigations*. OECD Competition Law and Policy Working Papers 311. Sept. 19, 2024. DOI: 10.1787/1ea7cdba-en. URL: https://www.oecd.org/en/publications/detecting-cartels-for-ex-officio-investigations_1ea7cdba-en.html (visited on 12/14/2025); OECD. *Digital Evidence Gathering in Cartel Investigations*. OECD Competition Law and Policy Working Papers 251. Aug. 5, 2020. DOI: 10.1787/95df5383-en. URL: https://www.oecd.org/en/publications/digital-evidence-gathering-in-cartel-investigations_95df5383-en.html (visited on 12/14/2025).

Korea/KFTC (Korea Fair Trade Commission)	BRIAS (Bid Rigging Indicator Analysis System)	Procurement System in order to screen for "red flags" Employs rules-based algorithms instead of deep learning
Singapore/Competition and Consumer Commission	BRDT (Bid Rigging Detection Tool) DST (Document Similarity Tool)	BRDT - Conducts statistical analysis of tender data to detect pricing anomalies and behavioral patterns indicative of cartel formation DST - Analyses tender documents to identify similarities that indicate cartel behavior (e.g. copy-pasting of sections)
Canada/Competition Bureau	COMPASS	LLM-based tool used to query documents and summarize evidence

The following section shall discuss the drawbacks of adopting systems that are mainly based on the connectionist approach to computational law (i.e. those reliant on machine learning/neural network models) It will then discuss the viability of a symbolic and hybrid approach.

Despite the tremendous financial investment and coverage in the popular press, there are indications that the case for GenAI productivity and performance may be overstated. According to a recent MIT study, despite the substantial investment in AI for the enterprise setting, 95% of organizations that have deployed generative AI are getting zero return.[3] Furthermore, only 2 out of 8 sectors show meaningful structural changes, seemingly disproving the claim of massive AI disruption. There are also more moderate estimates of AI macroeconomic effects: 20% of tasks in the US labor market is exposed to AI. However, only about a quarter of those tasks (5% overall) could be profitably

[3] Aditya Challapally et al. *The GenAI Divide - STATE OF AI IN BUSINESS 2025*. MIT, 2025. URL: https://mlq.ai/media/quarterly_decks/v0.1_State_of_AI_in_Business_2025_Report.pdf.

performed profitably after taking into account implementation costs.[4] Years after the hype of the Chat GPT launch, it is time to have a more sober discussion of the capabilities of these generative systems, in light of what we know about the inherent limitations of their internal mechanisms as well as their costs.[5]

9.1.2 Drawbacks of Generative AI

Lack of Grounding in Truth

Generative AIs such as LLMs are not aware of logical truth (or ground truth). For an LLM, all text (whether true or false) is by default treated equally.[6] Previous iterations of AI, based on supervised learning, linked the definition of performance with human-labeled data. Algorithms were guided by a "loss function" calculated based on distance between the algorithms prediction and an indisputable label provided by a human, thus providing some stable, objective basis for evaluation. In the case of generative AI, however, these models are trained via unsupervised learning on vast corpora of Internet data - largely untethered from human guidance. The objective functions of these models is the likelihood of the next token instead of closeness to the ground truth provided by human labels.

In the absence of explicit labeling, unsupervised learning models depend on the signals provided by Reinforcement Learning from Human Feedback (RLHF). For example, model designers can perform Supervised Fine Tuning, where the model is trained on a smaller, high-quality data set of human-written demonstrations (prompts and desired responses). Through this, the model learns

[4] Daron Acemoglu. "The Simple Macroeconomics of AI". in: *NBER WORKING PAPER SERIES* (May 2024). DOI: 10.3386/w32487. URL: https://www.nber.org/papers/w32487.

[5] Saba also points out that in the first place, it is difficult to come up with objective standards with which to evaluate the performance of LLMs, i.e. What does it mean for an LLM to have human-like performance? Technical benchmarks only capture a small portion of what humans actually do. Walid S. Saba. "Stochastic LLMs Do Not Understand Language: Towards Symbolic, Explainable and Ontologically Based LLMs". In: *Conceptual Modeling - 42nd International Conference, ER 2023, Lisbon, Portugal, November 6–9, 2023, Proceedings*. Ed. by João Paulo A. Almeida et al. Vol. 14320. Cham: Springer Nature Switzerland, 2023, pp. 3–19. ISBN: 978-3-031-47261-9 978-3-031-47262-6. DOI: 10.1007/978-3-031-47262-6_1. URL: https://link.springer.com/10.1007/978-3-031-47262-6_1 (visited on 04/16/2025), at 3.

[6] Saba, "Reinterpreting 'the Company a Word Keeps': Towards Explainable and Ontologically Grounded Language Models", at 2.

the structure and format of "helpful" responses.[7] Another mode of integrating human feedback is Proximal Policy Optimization, which optimizes not the output but the decision-making strategies (or "policies") of these unsupervised algorithms.[8] However, research from OpenAI's InstructGPT reveals that there is low agreement rate for human annotators (involved in RLHF), implying that human preferences is a poor proxy for ground truth - since this additional training data can be ambiguous, contested, or subjective.[9]. The persistent inability to connect to ground truth results in two related "behavioral" problems of generative AI systems - hallucinations and sycophancy.

Hallucinations A major stumbling block to the legal profession's embrace of these AI tools is the problem of hallucination. The term "AI hallucination" is still contested in the literature, although for most scholars it stands for any irrational behavior exhibited by artificial intelligence systems, particularly when the AI generates fictional, erroneous, or unsubstantiated information in response to queries.[10] In the context of legal applications, a hallucination[11] occurs when an LLM generates false or misleading information while presenting it as fact. This tendency originates from the model's primary function being next-token prediction (instead of relation to factual accuracy). If the data is

[7] See generally Long Ouyang et al. "Training Language Models to Follow Instructions with Human Feedback". In: 36th Conference on Neural Information Processing Systems. 2022.

[8] See Ouyang et al., "Training Language Models to Follow Instructions with Human Feedback"; See also Michał Oleszak. *Reinforcement Learning From Human Feedback (RLHF) For LLMs*. neptune.ai. Sept. 12, 2024. URL: https://neptune.ai/blog/reinforcement-learning-from-human-feedback-for-llms (visited on 12/07/2025), for an overview of Reward Modelling, where human labelers are given a prompt and multiple model outputs (for said prompt) and are asked to rank those responses. This produces a dataset of comparisons from which a reward model learns to optimize based on correlations with human rankings.

[9] Ouyang et al., "Training Language Models to Follow Instructions with Human Feedback".

[10] Yujie Sun et al. "AI Hallucination: Towards a Comprehensive Classification of Distorted Information in Artificial Intelligence-Generated Content". In: *Humanities and Social Sciences Communications* 11.1 (Sept. 27, 2024), p. 1278. ISSN: 2662-9992. DOI: 10.1057/s41599-024-03811-x. URL: https://www.nature.com/articles/s41599-024-03811-x (visited on 12/13/2025), at 1.

[11] The term "hallucination" as applied to AI is misleading - it implies that those systems are capable of perceiving the world, have an intentional mode of it, but the systems are only misrepresenting what they perceive. Hallucination, or even outright lying - assumes some concern with the truths of their statements. LLM's are not even capable of accurate representation (only the impression that they are doing so).What ChatGPT (and any similar system) is doing when it produces false claims is not lying and hallucinating (because either of these require some recognition of or concern for the truth). Instead, what it does is "bullshitting" in the philosophical sense put forward by Harry G. Frankfurt (On Bullshit, 2005). This is because these programs are incapable of being concerned with truth. Their design and training is oriented towards the production of statements that only seem true, without any concern for the truth. Michael Townsen Hicks, James Humphries, and Joe Slater. "ChatGPT Is Bullshit". In: *Ethics and Information Technology* 26.2 (June 2024), p. 38. ISSN: 1388-1957, 1572-8439. DOI: 10.1007/s10676-024-09775-5. URL: https://link.springer.com/10.1007/s10676-024-09775-5 (visited on 09/05/2025), at 1.

incomplete or the model cannot find a correct answer, the strategy shifts to inventing a plausible response[12]

The severity of the hallucination problem is neither trivial nor isolated. Research indicates that for general-purpose LLMs - that when asked direct verifiable questions about federal court cases, hallucination rates can range between 69 to 88 percent.[13] Furthermore, these errors are not random but pervasive, and can vary based on jurisdiction and legal issue. E.g. for housing law, hallucination rates can be as high as 64.9%. For employment law, it is lower at 44%.[14] Even if the AI is not providing hallucinated output, it may nevertheless present authentic legal materials as supportive of an argument when in fact, the same authority undermines the argument.[15] Another possibility is that the model may apply the authority to a problem that it was never meant to address. The problem has already manifested in legal research and advocacy. In *Mata v. Avianca*, plaintiff's attorney filed a submission before the court which contained citations of and quotes from non-existent cases. The court ordered the offending lawyers to pay a fine, stating that although using AI tools is not per se prohibited, lawyers bear a gatekeeping role in verifying their outputs.[16]

Retrieval Augmented Generation (RAG) is used to mitigate against hallucination. In a a RAG-enabled system, the model first retrieves relevant documents from a trusted database and then answers the prompt using only those retrieved documents. Even with RAG, however, it is still necessary to

[12] Annabel V. Teiling. *Will AI Render Lawyers Obsolete?* New York State Bar Association. Oct. 7, 2025. URL: https://nysba.org/will-ai-render-lawyers-obsolete/ (visited on 12/11/2025).

[13] Matthew Dahl et al. "Large Legal Fictions: Profiling Legal Hallucinations in Large Language Models". In: *Journal of Legal Analysis* 16.1 (Jan. 1, 2024), pp. 64–93. ISSN: 2161-7201, 1946-5319. DOI: 10.1093/jla/laae003. URL: https://academic.oup.com/jla/article/16/1/64/7699227 (visited on 12/11/2025).

[14] Damian Curran et al. "Place Matters: Comparing LLM Hallucination Rates for Place-Based Legal Queries". In: AI4A2J-ICAIL25: AI for Access to Justice at the International Conference on AI and Law 2025. Chicago, June 20, 2025. DOI: 10.48550/ARXIV.2511.06700. URL: https://arxiv.org/abs/2511.06700 (visited on 12/11/2025).

[15] Luca CM Melchionna. *Bias and Fairness in Artificial Intelligence - New York State Bar Association*. New York State Bar Association. June 29, 2023. URL: https://nysba.org/bias-and-fairness-in-artificial-intelligence/ (visited on 12/11/2025).

[16] See also *Gauthier v. Goodyear Tire & Rubber Co* (No. 1:23-CV-281 (E.D. Tex. Nov. 25, 2024)), where the court sanctioned attorneys who filed a response to a summary motion that contained citation to non-existent cases and several fabricated quotations. Court emphasized Rule 11, which requires filings to be grounded in existing law, and padded the onus on lawyers to "read and thereby confirm the existence and validity of, the legal authorities on which they rely" Judge Peter Kevin Castel. *Mata v. Avianca (Opinion and Order on Sanctions)*. June 22, 2023.

maintain a human in the loop - because the model can introduce subtle misinterpretations of the legal text.[17]

Sycophancy Sycophancy is the tendency of an AI model to align its responses with the user's stated or implied beliefs, even when those beliefs are factually incorrect.[18] AI models can be optimized for being agreeable to human users, and it may be possible to "jailbreak" models out of their safety concerns through prompts that exploit this tendency.

Problems such as hallucinations and sycophancy may be parts of a larger problem with alignment which is emergent with parameter size training data volume, and inconsistency of human feedback. Thus, simply feeding these systems more of the same data is not likely to solve these problems. The lack of ground truth is a barrier to adoption for a professional setting like law. In the context of competition law, for example, a regulator cannot deploy a chatbot that is undergirded by a generic dataset - it would need to be based on laws specific to their jurisdiction and community of practice. In a regulatory or competition law setting, given the high stakes involved, an agency can ill-afford to base its determinations on an opaque probabilistic model. Factual determinations will require not just textual correlations but some clear connections to both established facts and norms.

Bias and Toxicity

Connectionist AI systems (despite the computational, mathematical basis of their operation) should not be treated as objective or neutral. Bias can be embedded into these systems through: 1. The source/nature of the data they are trained on; 2. Assumptions of their programmers; 3. The incentives of the entities that display and monetize these models. The size and provenance of the training data does not guarantee diversity or mitigate bias. Bender cites instances when user-generated content favored in the training of these systems most likely overrepresent men and

[17] Dahl et al., "Large Legal Fictions".

[18] Mrinank Sharma et al. "Towards Understanding Sycophancy in Language Models". In: ICLR 2026. 2024. URL: https://doi.org/10.48550/arXiv.2310.13548.

underrepresent women.[19] The literature to date documents the forms of bias exhibited by LLMs, which could include:[20]

1. Stereotypical associations;
2. Negative sentiment towards specific groups;
3. Skewed perspectives on some controversial topics.

These tendencies may be attributable to the source of the training data, i.e. basing the training data on a raw intake of the open internet is likely to "overrepresent hegemonic viewpoints and encode biases against marginalized populations".[21] Thus, the way these sources are structured and maintained makes it likely that dominant viewpoints are privileged.[22]

The embedded bias in these systems violate the norms expected to be followed by decisionmakers in a legal setting (such as judges and regulators) - First, constitutional regimes enshrine the expectation that their decisions are rational and evidence-based. Second is the requirement that these decisions are not discriminatory.

Capability Limitations

Knowlege Barrier A generative AI model's knowledge is static, locked at the moment its training run concludes. It also lacks the the capability to access new information in real time.[23] Legal scholars seem to assume that AI models would have unlimited capacity to learn and adopt, when

[19] Emily M. Bender et al. "On the Dangers of Stochastic Parrots: Can Language Models Be Too Big?" In: *Proceedings of the 2021 ACM Conference on Fairness, Accountability, and Transparency*. FAccT '21: 2021 ACM Conference on Fairness, Accountability, and Transparency. Virtual Event Canada: ACM, Mar. 3, 2021, pp. 610–623. ISBN: 978-1-4503-8309-7. DOI: 10.1145/3442188.3445922. URL: https://dl.acm.org/doi/10.1145/3442188.3445922 (visited on 02/08/2023), at 613.

[20] Ibid., at 614.

[21] Ibid., at 610.

[22] Ibid., at 614.

[23] Retrieval Augmented Generatio (RAG) attempts to address this, but with a limited context.

these systems, as analogues of human minds, may simply be reiterating our limitations[24]. Machine learning occurs when two conditions are satisfied:[25]

1. Learnable regularity - Means that a machine can observe a pattern, perhaps when data is big enough or a model is expertly specified.

2. Contextual invariance - The pattern observed must repeat itself in a sufficiently similar context. A change in circumstances prevents learning.

These systems are based on existing data, and what they can extrapolate from it. Even assuming that such systems can scale up with enough data and computational power to completely capture the past, these will not be able to account for change and new patterns of behavior.[26] The ability to perform in games as well as well as generate code occurs within environments with fixed rules. On the other hand, the law can evolve in unexpected ways.[27] The way models are developed and trained (source of training data, cadence of training sessions due to financial costs) may also mean that it is relatively static and cannot take into account evolving social norms and language use.[28] The problem is of special concern for a field such as law. New laws, interpretations are constantly being enacted, old laws are made defunct at a rate that outpaces the cadence of training runs.

Reasoning Deficits Since LLMs rely on surface correlations in the training data instead of an accurate, up-to-date model of the world, they struggle with multi-step deductive reasoning, especially if the query requires them to connect disparate pieces of information, especially when the relationship between such information is neither in the prompt nor the immediate context window. LLMs convert chunks of text into high-dimensional vectors (embeddings). In effect, it "flattens" the subject from a

[24] Frank Fagan. "The Un-Modeled World: Law and the Limits of Machine Learning". In: *MIT Computational Law Report* (2022). ISSN: 1556-5068. DOI: 10.2139/ssrn.4189673. URL: https://law.mit.edu/pub/the-un-modeled-world/ (visited on 07/08/2024), at 2.

[25] Ibid., at 2.

[26] Ibid., at 3.

[27] Ibid., at 3.

[28] Bender et al., "On the Dangers of Stochastic Parrots".

set of rich granular entities into a list of numbers - discarding the relational semantics that would define a problem.

LLMs are based on an extensional paradigm - neural networks that can only deal with numerical values. But a lot of reasoning require conceptual/intensional definitions. LLMs cannot account for the intensional meaning of objects of cognition, and can't make good inferences in context that differentiate between intensional and extensional definitions.[29] Thus, LLMs can struggle with complex linguistic reasoning - a critical fault when applying them to a field such as law, which is deeply reliant on the interpretation and transformation of language.[30] It is not enough that the language produced is plausible, since legal outcomes can turn on getting complex linguistic phenomena entirely right (or not at all).

Opacity, Lack of Explainability

The explosive growth in model size and complexity also exacerbates the difficulty of understanding the training data.[31] To the extent that explainability requires "inference in reverse", that is, being able to step through the reasoning of a system, this can only be done practically through:

1. The invertibility of computation

2. Using a symbolic structure to describe a semantic map of the computation

Neither is this possible for generative AI, which operates at the sub-symbolic level and involves a stochastic element.[32] The prevalent computational approach to compliance checking of bulk document/transactions is based on Machine Learning techniques. Machine learning is difficult to employ for cases that require specific and exact values (which is usually required for due diligence)

[29] Saba, "Stochastic LLMs Do Not Understand Language", at 4-5.

[30] Lawyers regularly resolve quantifier scope ambiguity to translate natural language to more precise rules. This requires inferences from commonsense knowledge instead of context. E.g. For the statement "Every senator in a key electoral state voted for the amendment" The LLM usually fails to resolve the quantifier ambiguity correctly, most likely reading "a" to mean there is only one key electoral state. When in fact, it is $\forall senator \exists state$ ibid., at 6-7.

[31] Bender et al., "On the Dangers of Stochastic Parrots", at 610.

[32] Saba, "Stochastic LLMs Do Not Understand Language", at 1-2.

as well as drawing inferences from these values. As such, these systems often follows the Pareto principle. It handles the first 80% correctly. But due to its black box nature and consequently its lack of explainability, it is difficult to distinguish the 80% correct ones from the rest. It is also not explainable due to its black box nature.[33]Machine learning is inadequate due to its statistical instead of logical basis.

Costs

Financial Costs Consumer facing costs (usually expressed as subscription prices paid by users) may not reflect the true costs of running these systems, because user subscriptions are actually subsidized. In addition to the cost of compute from the user side, the development and deployment of AI represents a level of financial investment that makes it beyond the reach of all but the largest governments and companies. This may include the capital expenditure for data centers (land, buildings, and permits), development and compute costs for model training, as well as the inference costs for every query to the model.

New generative AI models are much more flexible, require less human annotation but require ever larger data sets and more computing power to crunch that data (training). Computing requirements increase with an order of magnitude (10x) with each new generation of models, with the associated computing costs rising proportionally. The estimated training costs was $1000 dollars for the first transformer model in 2017, to $120 million in training costs (in the case of the Gemini Ultra model) as of 2023.[34]. Cottier (2025) provides detailed estimates of the cost structure for the training of "frontier" AI models.[35] Training costs include the costs for hardware (GPUs and TPUs); the

[33]Anim refers to other compliance checking systems (i.e. one used for VAT compliance, from Lahann, et al) which use machine learning. Joseph K. Anim, Livio Robaldo, and Adam Wyner. "Compliance Checking in the Energy Domain via W3C Standards". In: *New Frontiers in Artificial Intelligence*. Ed. by Mayumi Bono et al. Vol. 14644. Cham: Springer Nature Switzerland, 2024, pp. 3–18. ISBN: 978-3-031-60510-9 978-3-031-60511-6. DOI: 10.1007/978-3-031-60511-6_1. URL: https://link.springer.com/10.1007/978-3-031-60511-6_1 (visited on 06/12/2025), at 134-135.

[34]Bertin Martens. "The Tension Between Exploding AI Investment Costs and Slow Productivity Growth". In: *Bruegel Working Paper Series* 2024.18 (Oct. 9, 2024), pp. 1–14. URL: https://www.bruegel.org/sites/default/files/2024-10/WP%2018%202024.pdf (visited on 12/02/2025).

[35]Frontier models are defined as models within the top 10 most compute intensive models upon release Ben Cottier et al. *The Rising Costs of Training Frontier AI Models*. Feb. 7, 2025. DOI: 10.48550/arXiv.2405.21015. arXiv:

energy consumed (or cloud compute costs); the model development cost (including R&D staff costs).[36] Assuming the current trend continues, then frontier models could cost as much as 1 billion dollars to train by 2027.[37]

The breadth of capability of generative AI comes at a hidden cost. It requires the training from a tremendous amount of information, much of which may be protected by intellectual property rights. Another component of the financial cost of these generative AI systems is the potential liability to IP rightsholders, for protected works included in the training data. The legal ground for such extraction is still being contested - both as a question of legal interpretation for judiciaries, as well as a policy matter for legislatures. Until a clear rule for the allocation of rights is settled, potential intellectual property liability should be factored into any reasonable cost estimate of AI deployment.[38] Generative AI may also face liability for hallucinations and misalignment. These liability costs are no longer merely theoretical. See *Mata v. Avianca*, discussed above, as well as the *Air Canada* case[39] case where the airline was held liable for a hallucinated refund policy, which gave rise to damages.

The price tag of generative AI will also have long-term structural effects: With the prohibitive cost of even just development (even prior to training runs), frontier models will only be accessible to the largest companies and government institutions. This may lead to a concerning concentration of power.[40] This gives rise to issues of equitable access to these models. Even if we can get access to the source code, as well as the data, the compute required for training these models (as well as the inference costs for any at-scale deployments) would be beyond most companies and governments.[41]

2405.21015 [cs]. URL: http://arxiv.org/abs/2405.21015 (visited on 12/02/2025). Pre-published, at 1.

[36] Cottier et al., *The Rising Costs of Training Frontier AI Models*, at 3.

[37] Ibid., at 6.

[38] See for example the $1.5 billion settlement between authors and Anthropic, which is estimated to be the largest copyright settlement in the U.S. to date. Dave Hansen. *The Bartz v. Anthropic Settlement: Understanding America's Largest Copyright Settlement*. Kluwer Copyright Blog. Nov. 10, 2025. URL: https://legalblogs.wolterskluwer.com/copyright-blog/the-bartz-v-anthropic-settlement-understanding-americas-largest-copyright-settlement/ (visited on 12/09/2025).

[39] *Moffatt v. Air Canada*, 2024 BCCRT 149

[40] Cottier et al., *The Rising Costs of Training Frontier AI Models*, at 10.

[41] Bender et al., "On the Dangers of Stochastic Parrots", at 611.

Environmental Costs Bender highlights the environmental impacts of the growth of Large Language Models, and urges the need to have a better accounting, not just of performance or financial costs, but also the amount of resources consumed by these systems.[42] The environmental costs of these AI models in terms of energy and CO2 emissions are tremendous. For example, training a single instance of BERT, an early transformer model, would require as much energy as a trans-American flight.[43] Assuming the current rate of AI-related buildout is sustained, the technology is estimated to account for annual water footprint ranging from 731 to 1,125 million cubic meters and additional annual carbon emissions from 24 to 44 metric tons of CO2 by 2030.[44]

The environmental impacts are disproportionate, since they are most likely to affect marginalized communities (who in turn are not likely to receive any benefits from the growing capabilities of these systems), but are the same time are the most vulnerable from the negative environmental impacts caused by the expansion of these systems.[45] We are already seeing some indications of communities suffering from water shortages, environmental degradation, direct harm from being the site of data centers.[46]

Effects on the Legal Profession

The work of the legal profession is often characterized by complex reasoning, precision in language, and the ability to interpret a broad corpus of interconnected legal texts. These high-level cognitive requirements, coupled with high barriers to entry gave rise to the assumption that it would be immune from displacement by technology. The historical trend has been that technological

[42] Ibid., at 610.

[43] Ibid., at 612.

[44] Tianqi Xiao et al. "Environmental Impact and Net-Zero Pathways for Sustainable Artificial Intelligence Servers in the USA". in: *Nature Sustainability* (Nov. 10, 2025). ISSN: 2398-9629. DOI: 10.1038/s41893-025-01681-y. URL: https://www.nature.com/articles/s41893-025-01681-y (visited on 12/08/2025).

[45] Bender et al., "On the Dangers of Stochastic Parrots", at 610.

[46] For an overview of the energy and water footprint based on current trends Xiao et al., "Environmental Impact and Net-Zero Pathways for Sustainable Artificial Intelligence Servers in the USA"; For a specific case of community impact, see Josh Saul et al. *AI Data Centers Are Sending Power Bills Soaring*. Bloomberg.com. Sept. 29, 2025. URL: https://www.bloomberg.com/graphics/2025-ai-data-centers-electricity-prices/ (visited on 12/08/2025); See also for a perspective from the Global South Nikhil Inamdar. *India's Data Centre Boom Confronts a Looming Water Challenge*. BBC. Nov. 9, 2025. URL: https://www.bbc.com/news/articles/cgr417pwek7o (visited on 12/08/2025).

development through automation has disproportionately affected low-wage, manual, routine labor (so-called "blue-collar" work) The current explosion of AI, on the other hand, inverts this historical trend and could affect non-routine cognitive tasks performed by highly-educated wage earners ("white-collar" work).[47] The International Monetary Fund (IMF) estimates that 40 percent of workers worldwide are in high-exposure occupations (with advanced economies more exposed).[48] For US workforce: 80% pf US workforce could have at least 10% of their work tasks affected by the introduction of LLMs, and 19% of workers may see at least 50% of their tasks impacted.[49] Research from the University of Pennsylvania and Open AI indicates that educated white-collar workers earning up to $80,000 annually are among the most exposed to workforce automation.[50]

The legal profession may be especially vulnerable, since many of its tasks appear to align well with the capabilities of LLM's - manipulation of language, synthesis of precedents the prediction of outcomes, and the generation of structured text. Lawyers have long used AI in their workflows particularly through predictive AI tools such as Lex Machina and Westlaw Edge. These tools analyzed structured data (e.g. dockets and case outcomes) in order to forecast judicial behavior or case timing.These tools, however, were limited to metadata about the work product (as opposed to the work product itself). These provided strategic intelligence through probabilities and had very little direct participation in the creation of the work product.[51] On the other hand, tools based

[47] See National Academies of Sciences, Engineering, and Medicine. *Artificial Intelligence and the Future of Work*. Washington, D.C.: National Academies Press, Apr. 10, 2025, p. 27644. ISBN: 978-0-309-71714-4. DOI: 10.17226/27644. URL: https://nap.nationalacademies.org/catalog/27644 (visited on 09/24/2025); See also Mauro Cazzaniga et al. "Gen-AI: Artificial Intelligence and the Future of Work". In: *IMF Staff Discussion Notes* (SDN2024/001 Jan. 2024). URL: https://www.imf.org/en/-/media/files/publications/sdn/2024/english/sdnea2024001.pdf, "Unlike previous automation waves, which affected mostly middle-skilled workers, AI's displacement risks span the entire income spectrum, including high income earners and skilled professionals...However, the potential for AI to complement jobs is positively correlated with income levels...With strong complementarity, high-wage earners might experience a disproportionate increase in their earnings".

[48] Cazzaniga et al., "Gen-AI: Artificial Intelligence and the Future of Work"; Exposure is defined as a measure of whether access to an LLM or LLM-powered system would reduce the time required for a human to perform a specific detailed work activity or complete a task at least 50 percent. Tyna Eloundou et al. "GPTs Are GPTs: An Early Look at the Labor Market Impact Potential of Large Language Models". In: *Science* 384.6702 (June 20, 2024), pp. 1306–1308. DOI: 10.1126/science.adj0998.

[49] Eloundou et al., "GPTs Are GPTs".

[50] Ibid., at 6-7.

[51] Mike Robinson. *Law Firm Predictive Analytics: Use Cases And Applications*. Clio. Apr. 17, 2025. URL: https://www.clio.com/blog/law-firm-predictive-analytics/ (visited on 12/10/2025).

on LLMs have the ability to generate work product such as complex brief, summarize discovery materials, and conduct natural language legal research.[52]

Complementarity While exposure to AI automation risk will not necessarily lead to immediate job loss - it means that tasks performed by exposed workers can be augmented or performed by AI. These systems can be complementary to workers with relevant judgment and training.[53] Professionals such as lawyers and judges, although they have high exposure to AI, can harness it as supporting tools, and therefore have a high degree of complementarity.[54]Although more in-depth long term studies are needed, some sources suggest a degree of complementarity between the use of generative AI and legal work, at least for some tasks such as discovery, contract review, legal research and drafting.[55]

Business Model Friction The adoption of AI may create short term friction with the dominant business model of the legal profession, which is centered around the generation of billable hours. If, for example, the use of an AI tool allows an associate to draft a contract in 2 hours instead of 10, a firm whose business model is built on billable hours will be hesitant to use the AI tool.[56] The decrease in raw number of hours however, may be compensated by the increased quality of those

[52]Michael Chui et al. *The Economic Potential of Generative AI: The next Productivity Frontier*. McKinsey & Company, June 2023. URL: https://www.mckinsey.com/~/media/mckinsey/business%20functions/mckinsey%20digital/our%20insights/the%20economic%20potential%20of%20generative%20ai%20the%20next%20productivity%20frontier/the-economic-potential-of-generative-ai-the-next-productivity-frontier.pdf.

[53]If AI completely supplants the need for expertise, then there is complementarity. If AI supplements expertise but does not obviate the need for it, there is complementarity. One indicator of complementarity: If human performance increases when supplemented with AI, then there is positive complementarity. National Academies of Sciences, Engineering, and Medicine, *Artificial Intelligence and the Future of Work*, at 125.

[54]Carlo Pizzinelli. "Labor Market Exposure to AI: Cross-country Differences and Distributional Implications". In: *IMF Working Papers* 2023.216 (Oct. 2023), p. 1. ISSN: 1018-5941. DOI: 10.5089/9798400254802.001. URL: https://elibrary.imf.org/openurl?genre=journal&issn=1018-5941&volume=2023&issue=216&cid=539656-com-dsp-crossref (visited on 08/05/2024); National Academies of Sciences, Engineering, and Medicine, *Artificial Intelligence and the Future of Work*, at 125-126.

[55]See Thomson Reuters. *2025 Generative AI in Professional Services Report*. 2025; Niki Black. *The Legal Industry Report 2025*. Federal Bar Association. Apr. 22, 2025. URL: https://www.fedbar.org/blog/the-legal-industry-report-2025/ (visited on 12/14/2025).

[56]Richter(2025) estimates that 43% of legal professionals foresee a decline in hourly billings over the next five years due to the efficiency gains from AI. Marjorie Richter J.D. *How AI Is Transforming the Legal Profession*. Thomson Reuters Law Blog. Aug. 18, 2025. URL: https://legal.thomsonreuters.com/blog/how-ai-is-transforming-the-legal-profession/ (visited on 12/11/2025).

hours and the faster, more accurate work produced. This will result in billings that are less likely to be challenged and written down by the clients.[57]

Career Ladder Collapse One immediate effect, already evidenced in recent trends, is the contraction of junior associate hiring. With AI handling the more routine, labor-intensive tasks (e.g. document review, basic research, initial drafting) previously the primary tasks of entry level associates, firms do not have to hire as many novices. However, handing off most of the "grunt work" to AI will not only result in fewer associates, those associates that are hired will be overdependent on an "AI crutch" and lose touch with the law at an in-depth, granular level.[58] These might lead to long-term changes to hiring practices and firm structures: Firms will compete over mid-level and senior talent (with the contextual experience and technological competence) to review AI generated work. However, with a hollowed-out training pipeline for new lawyers, there might be fewer such senior associates.

Impact on Governance

The adoption of generative AI systems within a government system can surface the more sinister tendencies of what Scott calls the synoptic view.[59] A central premise of Scott's account is that the state is usually not interested (or is in fact adverse to) the richness and organizational complexity of human lives (which is what it should ultimately strive to protect). The state develops the viewpoint that nature and people as resources to be organized in order to maximize efficiency of extraction.[60] The move towards legibility is responsible for the modern administrative state - Certain things

[57] Robert Couture. *The Impact of Artificial Intelligence on Law Firms' Business Models*. Harvard Law School Center on the Legal Profession. Feb. 24, 2025. URL: https://clp.law.harvard.edu/knowledge-hub/insights/the-impact-of-artificial-intelligence-on-law-law-firms-business-models/ (visited on 12/11/2025).

[58] Teiling, *Will AI Render Lawyers Obsolete?*

[59] This proceeds from the account that the pre-modern state was "blind", without usable awareness of its people and their environment. Without this awareness it could not make "rational", fine-grained governance decisions. James C. Scott. *Seeing Like a State: How Certain Schemes to Improve the Human Condition Have Failed*. Yale Agrarian Studies. New Haven, London: Yale University Press, 2020. 445 pp. ISBN: 978-0-300-07016-3 978-0-300-24675-9, at 2.

[60] For this point, Scott cites the phenomenon of modern forestry which arose from Germany's need to maximize tax revenue. This function narrowed relevant variables to acreage, yield, and market value. Wild forests (with resilience and diversity) were illegible to the bureaucrat trying to make sense along the statist variables. Thus, the need to reshape forests to be more legible. ibid., at 11-12.

that we take for granted, like last names, standard weights and measures, census, land registration, even the design of cities - are the state's attempts to make the people and the environment more "legible". That is, to convert otherwise complex social practices into a standardized practices into a standardized scheme that can be recorded and monitored. Improvements in the legibility helped the state carry out basic functions, like taxation, conscription, but also public health, political surveillance, relief for the poor (and a host of other interventions, both harmful and benign)[61] Nevertheless, if not counterbalanced with more humanistic tendencies, state projects that run along this mode (rationalizing, legible) have often led to fiascoes - particularly in the 3rd world. However well intentioned, they have resulted in economic damage, and lives lost.[62]

As AI becomes more ubiquitous, it accelerates the long-running trend of the disappearance of the line between social and technological orders.[63] Software has manifested and amplified the statist tendency for building the synoptic view through reduction and abstraction. Both simple programs and advanced AI models view messy phenomena as capable of being represented as data structures that are amenable to organization. Transformation into data structures enable indirection or disruption - breaking an existing socio-technical process, replacing their components with abstractions.[64]

The same statist logic of organization may also be at work for the enterprises that have monopolized the design and at-scale deployment of these systems. They are not necessarily designing these systems to preserve the long-term viability of legal reasoning of specific systems. They need things to be flat and neutral. Although AI systems are more complicated than traditional software, they still embody the same logic of abstraction and reduction.[65] The training data is scraped from

[61] Ibid., at 2.

[62] Ibid., at 3.

[63] Barath Raghavan and Bruce Schneier. *Seeing Like a Data Structure*. Harvard Kennedy School Belfer Center for Science and International Affairs. May 23, 2024. URL: https://www.belfercenter.org/publication/seeing-data-structure (visited on 08/13/2024).

[64] Platforms demonstrate this tendency to abstract and flatten: Food delivery apps transform a city's rich restaurant scene into a consolidated ghost kitchens; Social interactions are reduced to the metric of "engagement"; ibid., at 4-5.

[65] Ibid., at 12-13.

the Internet, which is itself a flattened representation of human knowledge and interaction. The model architectures are designed to process text as sequences of tokens, without regard for the underlying meaning or context. The optimization process is focused on statistical patterns, rather than understanding or reasoning. Thus, these systems are not designed to preserve the richness and complexity of legal reasoning, but rather to reduce it to a form that can be processed by the model. The complexities of research, communication, and decision-making are flattened into patterns of prompt engineering and model tuning.

9.1.3 Integration of Symbolic Approaches into Generative AI

As the previous discussion demonstrates, the alignment of AI with the legal environment (in terms of costs, recognition of norms, etc.) is not spontaneous. Compliance has to be engendered across all phases of these systems' lifecyles: from design to training until maintenance.[66] The size, cost, and opacity of generative AI systems, however, converge to make this infeasible.

Rules-based systems based on symbolic processing - with explicit declarations of concepts and norms, provided the original approach for computational law. Although successful for some domains, and now quietly integrated in some enterprise systems, they have not been funded and deployed at the same scale as the current batch of generative AIs. Rule-based systems such as those formally declared knowledge graphs are not as performant, since most real-world problems cannot be fully anticipated and modeled by knowledge graphs and their constraints. This limitation can be mitigated by:

1. Tempering our expectations, using these systems for more constrained domains more susceptible to a rules based approach, and then using them not as the basis for automated decisions, but only as basis for initiating formal inquiries;

[66] Ozana Olariu. "Human Rights Compliant Artificial Intelligence in the Administration of Justice". In: Convention for the Creation of a Digital Legal Framework for the Social Progress of the Philippines. University of Granada, Oct. 15, 2025.

2. Another possibility is combining rules-based systems with generative AI. This will give us the breadth and expressiveness of connectionist models with the rigor of rules-based systems.

There is promising work in a hybrid approach where the expressiveness of generative AI systems are combined with external verifiers (via RAG) and formal logical constraints (Constitutional AI). As more systems are handed off to AI there is a need to impose firm "constitutional" constraints.[67] An initial approach to "Constitutional AI", which involves training a model to align with human values, by training it to follow a predefined set of ethical principles. These principles form a "constitution" and guide the model towards human-preferred tendencies, such as helpfulness, harmlessness, and honesty. This has been the approach taken by Anthropic for its Claude LLM, which draws its constitution from the UN Declaration of Human Rights, as well as best practices for trust and safety collated from other AI labs.[68] Since the guidance in this constitutional AI paradigm is encoded in natural language, however, it remains subject to the same stochastic process as the LLM, and the model's adherence can be probabilistic and subject to drift. A natural language expression of a norm can be ambiguous. The principle of "harmlessness" can conflict with "helpfulness" in nuanced ways, and the probabilistic resolution of this conflict can lead either to evasiveness (refusing to answer safe questions) or sycophancy (agreeing with user bias).

Neuro-Symbolic AI refers to AI systems that seek to integrate neural network based methods with symbolic knowledge-based approaches.In the context of providing constitutional constraints to generative AI, this involves formalizing norms not as natural language statements but as structured data.[69] Neuro-Symbolic AI represents the convergence between the two main branches of AI development: the connectionist approach, with its statistical learning power with the rigor and

[67] Stephen Wolfram. *Computational Law, Symbolic Discourse and the AI Constitution*. Stephen Wolfram Writings. Oct. 12, 2016. URL: https://writings.stephenwolfram.com/2016/10/computational-law-symbolic-discourse-and-the-ai-constitution/ (visited on 01/14/2024), at 169.

[68] Yuntao Bai et al. *Constitutional AI: Harmlessness from AI Feedback*. Dec. 15, 2022. DOI: 10.48550/arXiv.2212.08073. arXiv: 2212.08073 [cs]. URL: http://arxiv.org/abs/2212.08073 (visited on 12/15/2025). Pre-published.

[69] Amit Sheth, Kaushik Roy, and Manas Gaur. "Neurosymbolic Artificial Intelligence (Why, What, and How)". In: *IEEE Intelligent Systems* 38.3 (May–June 2023), pp. 56–62. DOI: 10.1109/MIS.2023.3268724. arXiv: 2305.00813 [cs]. URL: https://ieeexplore.ieee.org/document/10148662 (visited on 12/17/2025), at 1.

logical power of symbolic systems. For example: An ontology can provide a source of truth that is grounded in an established semantic. If a medical decision support system is confronted with a rule that prohibits drugs, it does not need to guess through context if "Ibuprofen" is a drug, since the ontology can define it as a `:Drug`. Similarly every decision made can be recorded as a traversal in the graph structure and made part of an audit trail, making explainability and corrections easier. A definition of legal/normative constraints in OWL and SHACL may help alignment (by providing structure and clear constitutional constraints) between the jurisdictions rules with LLMs.

Deployed by itself or in conjunction with generative AI, symbolic systems provide advantages in terms of lower total cost of ownership, better explainability, and complementarity with the legal profession.

Lower costs It may be difficult to compare overall costs because rules based systems were never deployed to the same level as generative AI. Nevertheless, comparing individual cost components can be indicative. Training and running AI models require specialized high performance Graphic Processing Units (GPUs) or Tensor Processing Units (TPUs). The graphs-based processing of the symbolic approach, however, will only require commodity CPUs and RAM, and tend to follow deflationary Moore's Law pricing curves. Initial startup costs for symbolic AI was high due to the "knowledge bottleneck" - it was slow and difficult to manually extract rules from a domains unstructured data. However, once the domain knowledge has been encoded, costs for maintaining it will be more manageable. Generative AI applications often feature a variable cost model based on the output expected from the model: Inference costs can easily scale up based on the nature of the query and the size of the user base. On the other hand, symbolic systems have a fixed cost, high throughput model. It is also more efficient - a single graph-based server can handle the same query load that would require an LLM a cluster of TPUs that is an order of magnitude more expensive. In order to correct or update an LLM model will require a new training run, which, as discussed above, can be prohibitively expensive. In contrast, an update for a symbolic system is a standard database operation with negligible costs. Finally, in terms of computational and energy costs - While

both training and inference are energy-intensive for generative AI, the graph traversals required for symbolic AI inference are highly optimized.

Explainability The symbolic approach, regardless of the actual form of encoding is more explainable because it is based on a formal logic that is legible to humans.[70] Even assuming that we can open up the black box of generative AI, the mechanism as exposed will only show us which weights in what layer were activated, and how the subsequent cascade of activations led to the output. In contrast to the black box nature of generative AI, the graph structures in a symbolic system have inherent traceability. Each traversal for the purpose of inference creates an audit trail.

Complementarity with the Legal Profession The use of symbolic systems contemplates a future where lawyers will be responsible for building and maintaining the underlying structure for "constitutional AI's". Under such a regime, lawyers can help structure and update that internal model to help make the models more reliable, truthful. This work will need to be maintained in the long term, as the law, norms and language continue to evolve, requiring new skills and perhaps new "legal technologist" positions in government and legal practice.

Limitations, Responses, and Future Work One limitation of a symbolic approach is the manual method through which the definitions and constraints are encoded.[71] Both the initial encoding and updating symbolic representations can take a lot of resources, especially time, spent mostly in manual work performed by experts.[72] Several strategies can be explored to mitigate this weakness, which can be explored in future work, all of which can leverage the ecosystem of tools that have grown around semantic web standards. For example, the natural language version of a laws and regulations can be enacted with a computational encoding, to be updated and elaborated as interpretive administrative issuances and court decisions are issued. Agents and platforms can be mandated to log the terms of their transactions into a computational form, which can be aggregated and compared to the

[70] Livio Robaldo et al. "Efficient Compliance Checking of RDF Data". In: *Journal of Logic and Computation* 33.8 (Dec. 2023), pp. 1753–1776, at 1754.

[71] Anim, Robaldo, and Wyner, "Compliance Checking in the Energy Domain via W3C Standards", at 135.

[72] Robaldo et al., "Efficient Compliance Checking of RDF Data", at 1754.

SHACL-defined constraints of the law. Finally, LLMs can be used to read the unstructured data (statutes, court decisions)to output an initial version of the structured graph data.

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